Dereham Church of England Junior Academy

Diocese of Norwich
Education and
Academies Trust

Calculations Policy

Mathematics is a beautiful subject which has its own unique place in the curriculum. It provides pupils with powerful ways to describe, analyse and problem solve in the world.

Pupils can experience a sense of awe and wonder as they solve a problem for the first time, discover a more elegant solution and make links between different areas of mathematics. Mathematics is the means of looking at the patterns that make up our world and the intricate and wonderful ways in which they are constructed and realised.

The language of mathematics is international. The subject transcends cultural boundaries and its importance is universally recognised. Mathematics helps us to understand and innovate the world.

Mathematics makes a significant contribution to modern society:

- the basic skills of mathematics are vital for the life opportunities of our children;
- mathematics develops the mind and those highly valued cognitive skills.

Our children study mathematics to become functioning adults who are able to think mathematically enabling them to reason and solve problems in a range of contexts.

"Good mathematics teaching is lively, engaging and involves a carefully planned blend of approaches that direct children's learning….the pitch and pace of the work is sensitive to the rate at which children learn while ensuring expectations are kept high and progress is made by all children"

This document aims to support:

- **The development of our children's fluency with basic number facts**
- **The development of our children's fluency in mental calculation**
- **The development of our children's fluency in the use of written methods**

Counting and Beginning Number Sense

SUBITIZING	COMPARISON	COUNTING	ONE-TO-ONE CORRESPONDENCE	CARDINALITY	HIERARCHICAL INCLUSION	NUMBER CONSERVATION
Being able to visually recognize a quantity of 5 or less	Being able to compare quantities by identifying which has more and which has less	The rote procedure of counting. Includes both verbal counting and object counting. The actual meaning attached to counting is developed through one-to-one correspondence	Being able to connect one number with object and then count each object with understanding	Begin able to tell how many objects are in a set—understanding that the last word in the counting sequence names the quantity for that set	Understanding numbers are nested inside each other and the number grows by one each count. For example, 3 is inside 4 or 4 is the same as 3+1 more	The number of objects remains the same when they are rearranged spatially. For example, 5 can be… 4 and 1 OR 3 and 2 OR 2 and 3, etc.

Development of abstraction and order irrelevance →

progression of development

Based on Clements, D.H. & Sarama, J. (2009) *Learning and teaching early math: The learning trajectories approach;* Van de Walle, J.A. & Lovin, L.H. (2006) *Teaching student-centered mathematics;* Fletcher, G. (2017) The progression of early number and counting. [Blog post]. Retrieved from https://gfletchy.com/2017/03/26/the-progression-of-early-number-and-counting/

What is Number Sense?

Number sense is an emerging construct that refers to a child's fluidity and flexibility with numbers and what numbers mean as well as an ability to perform mental mathematics and to look at the world and make comparisons.

There may be:

1. An awareness of the relationship between number and quantity
2. An understanding of number symbols, vocabulary and meaning
3. The ability to engage in systematic counting, including notions of cardinality and ordinality
4. An awareness of magnitude and comparisons between different magnitudes
5. An understanding of different representations of number
6. Competence with simple mathematical operations
7. An awareness of number patterns including recognising missing numbers

Number sense develops gradually over time and at different rates, through exploring numbers, visualising them in a variety of contexts, and relating them in ways that are not limited by formal written methods.

Number sense is the ability to be flexible with numbers and to understand both how our number system works and how numbers relate to each other. Children with good number sense have a range of mathematical strategies at their disposal and they know when to use them and how to adapt them to meet different situations.

Children at Dereham Church of England Junior Academy should receive regular opportunities to count to support building their number sense and automaticity.

Calculating Mentally

The ability to calculate in your head is an important part of mathematics. It is also an essential part of coping with society's demands and managing everyday events.

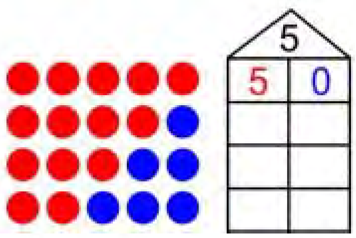
There is a need to review, consolidate and build on children's developing mental calculation skills throughout years 3 to 6.
Key areas to review in mental calculation with whole numbers, including the recall of number facts, and fractions, decimals and percentages, is set out in the our 'Basics in Number' document.

Further teaching guidance can be found in 'Teaching Children to Calculate Mentally'.

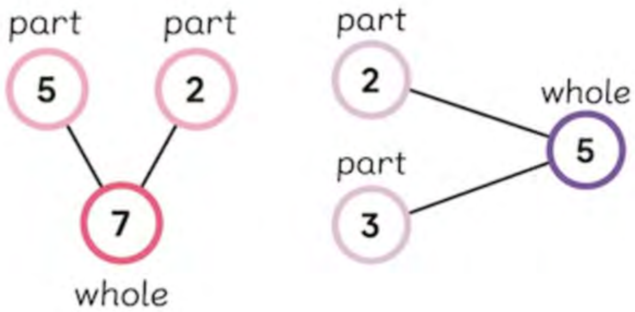
Number Bonds

Number bonds show how numbers are (partitioned) split or combined.
They let students split numbers in useful ways. They show how numbers join together, and how they break down into component parts. Number bonds forge the number sense needed for early primary students to move to addition and subtraction. As students progress, number bonds become an essential mental problem solving strategy.

Developing instant recall alongside conceptual understanding of number bonds to 10 is important. This can be supported through the use of images such as the example illustrated below:

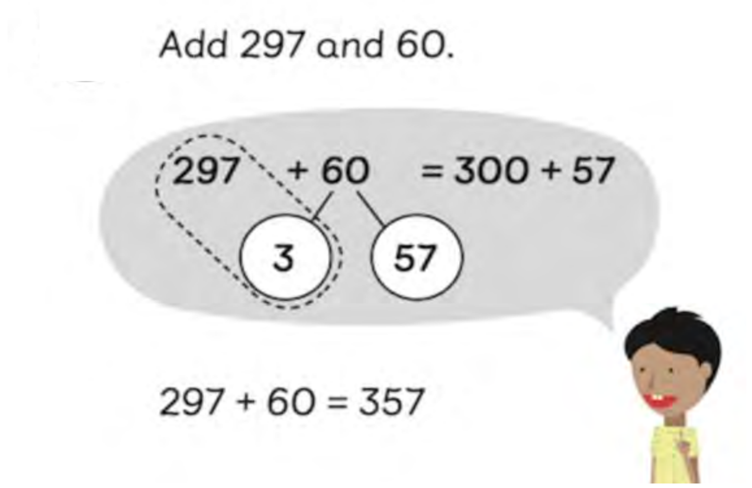


The image lends itself to seeing pattern and working systematically and children can connect one number fact to another and be certain when they have found all the bonds to 5.



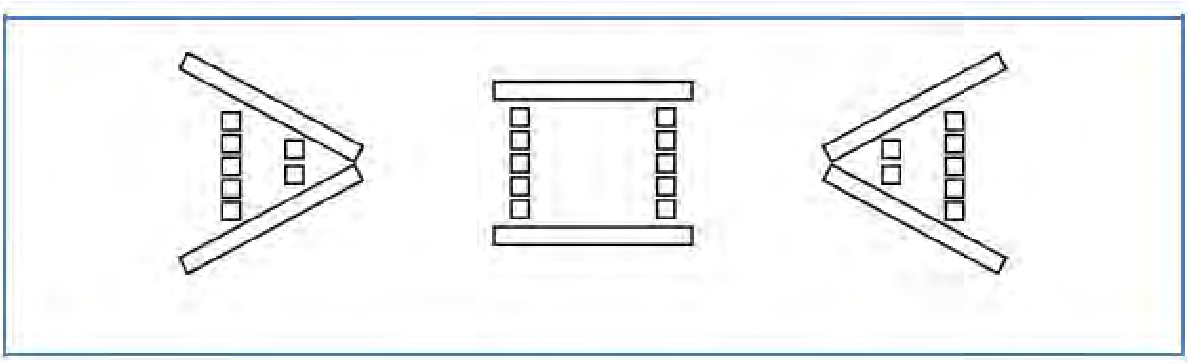
Early partitioning in a part-whole model

Using number bonds to support mental calculations.



Teach inequality alongside teaching equality

To help children develop their understanding of equality, they also need to develop understanding of inequality. One way to introduce the < and > signs is to use rods and cubes to make a concrete and visual representations such as:



to show that 5 is greater than 2 (5 > 2), 5 is equal to 5 (5 = 5), and 2 is less than 5 (2 < 5). Balance scales can also be used to represent inequality.

Vocabulary

Using correct mathematical language is crucial for thinking, learning and communicating mathematically. Children may build knowledge through remembering information that they hear, but it is only when they put these ideas into their own words that it becomes clear whether concepts have been learned effectively. It is in listening to children talking about mathematics that we, as teachers, can best assess what they are actually learning and understanding.

This enables us to identify and address any misconceptions that might be developing. We need to encourage children to explain what they are doing and why they are doing it. We must offer them opportunities to use mathematical language frequently, for example by participating in paired activities, group discussions and games as well as other dialogues. This will help children to learn new vocabulary, to use words they already know more accurately, and to express new ideas and new thinking.

It is important to encourage children not just to learn and remember the correct vocabulary, but also to use these words regularly to communicate mathematically. This will play a vital role in enabling children to develop their mathematical thinking, as appropriate use of mathematical language is essential for developing an argument or proof.

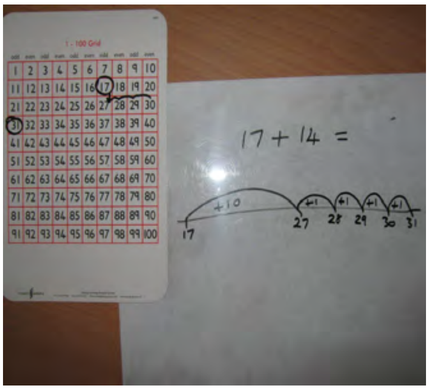
Using mathematical vocabulary can help all children to make links across areas of mathematics, across the curriculum as a whole and also within real-life situations. It can especially support lower attainers, enabling them to build confidence, communicate and problem solve, so should be an integral part of every mathematics lesson.

Addition

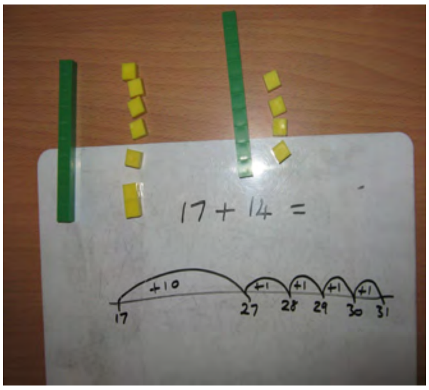
Partitioning on a number line

Partitioning to count on in multiples of 100, 10 or 1, is one of many examples which work well on a **number line.** Number lines will be used for calculations right through Key Stage 2 and beyond. This is a way of modelling mental calculation.

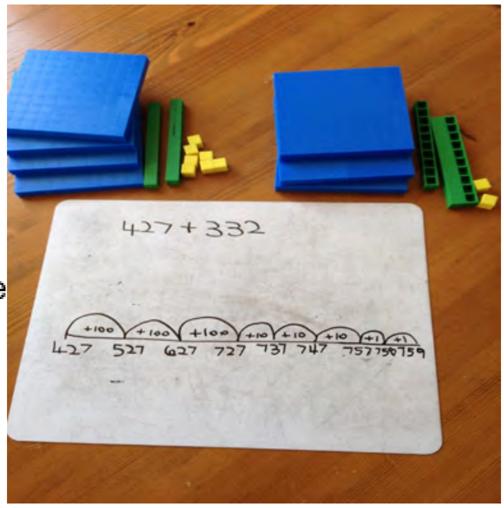
17 + 14

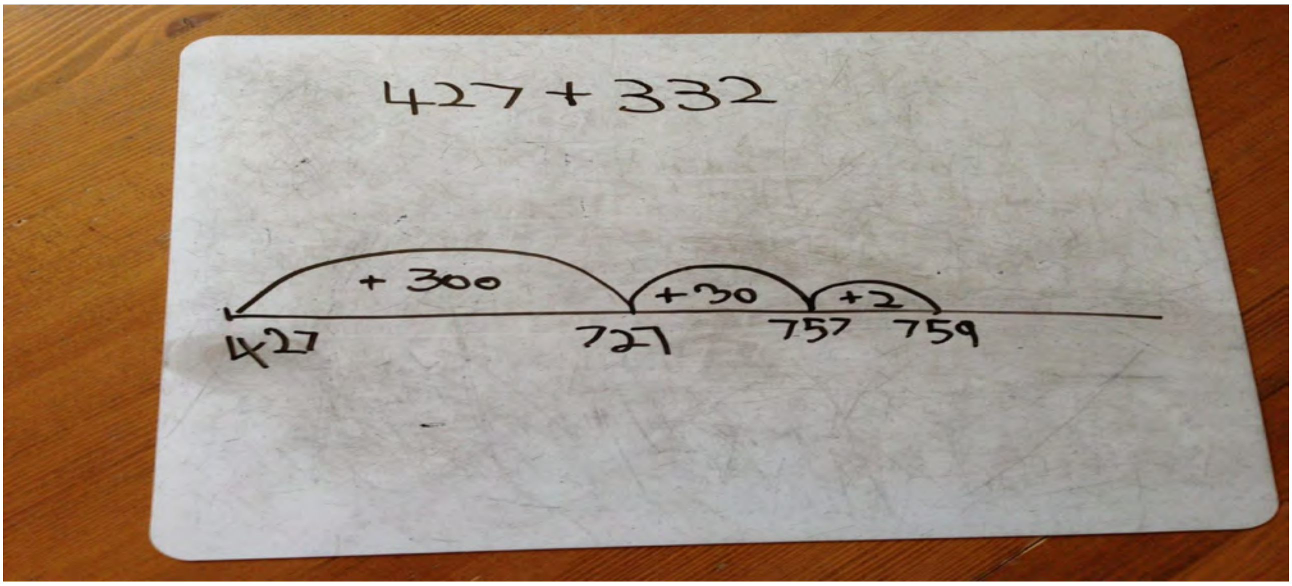


Initially, use resources such as a number square or base 10 blocks to aid the understanding of first adding the 10s and then adding the 1s.



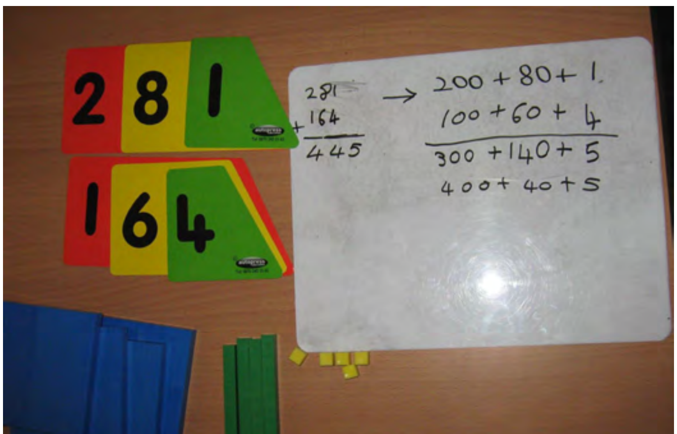
It is useful to use arrow cards to partition the numbers when beginning to work with larger numbers. Then separate the arrow cards and make each of the partitions using base 10. This gives the children a secure understanding of the value of each digit. When the children are secure with this, you may want to remove the arrow cards.

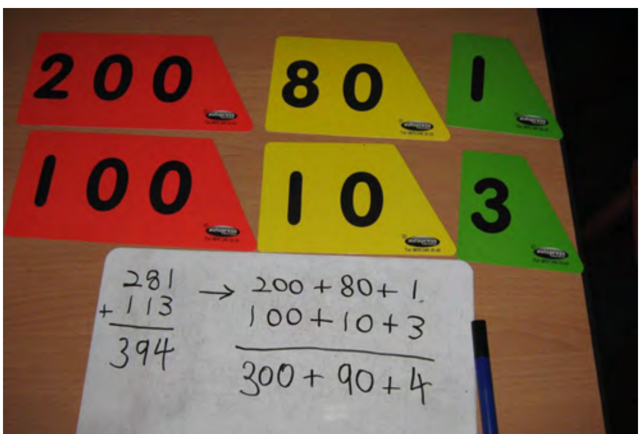




In year 3, pupils may begin to record their addition calculations vertically – adding numbers in columns. The vocabulary used when doing this will always be the whole number place value vocabulary. So, 54 would always be 50 (5 tens) and 4 (4 ones), never 5 and 4.

A way of transitioning from number line partitioning to column addition would be to vertically partition numbers. Again, the use of arrow cards and base 10 blocks cements the understanding and use of correct whole number place value vocabulary.

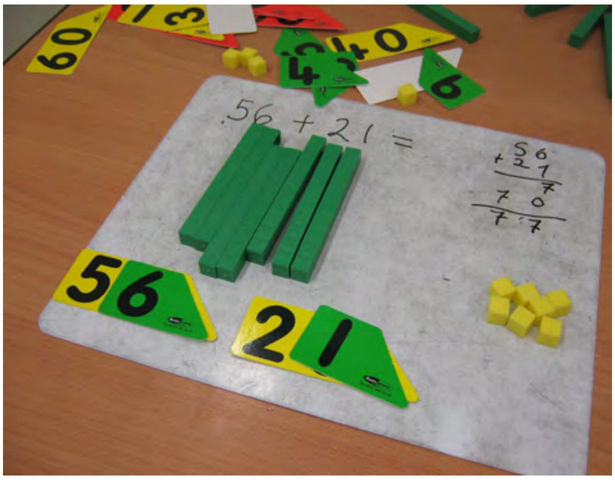


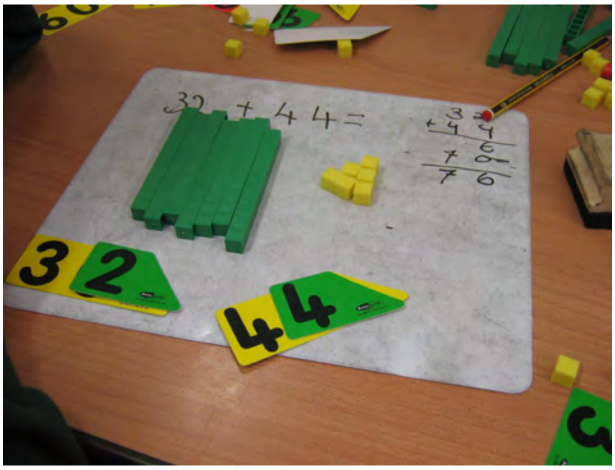


Expanded column addition

Once pupils have demonstrated understanding of place value, they can record their addition purely vertically.

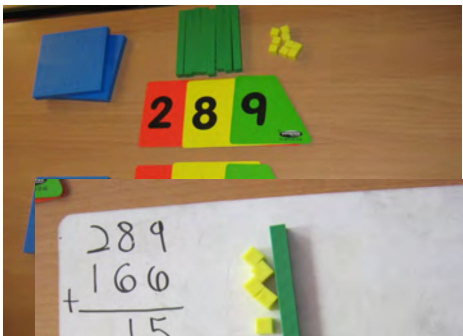
Arrow cards can be used to help the children record each step.



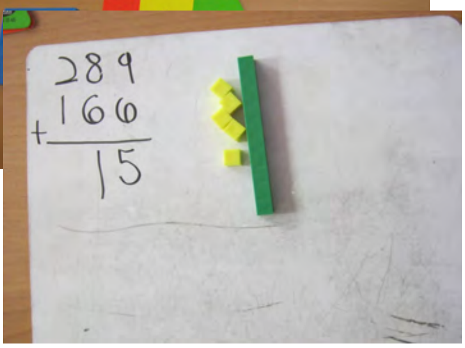


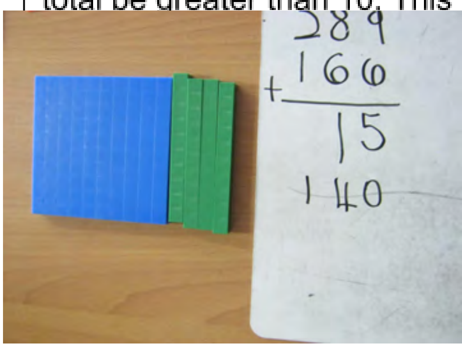
Once the pupils are very confident with this method of recording they may extend it to 3-digit numbers, providing their understanding of place value is sufficient to support this.

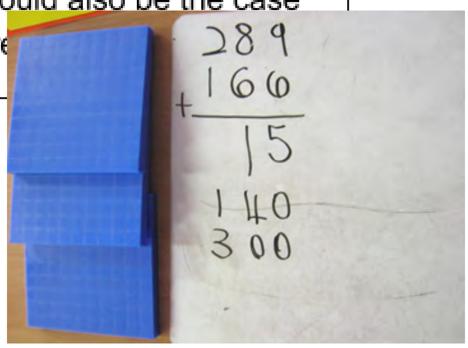
Understanding that addition in commutative means that hundreds do not have to be added first as any order of adding will give the same total. When adding vertically, children should begin with one's.



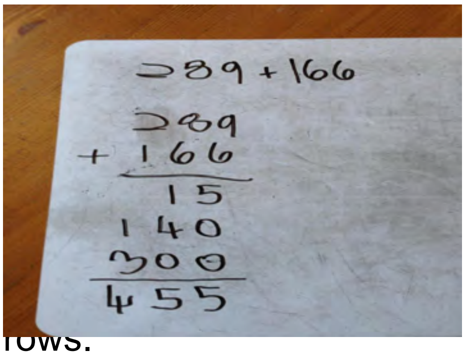
Once the base 10 blocks have been collected for the 2 numbers, the children may find it easier to exchange the ones cubes for a 10 stick should the total be greater than 10. This could also be the case



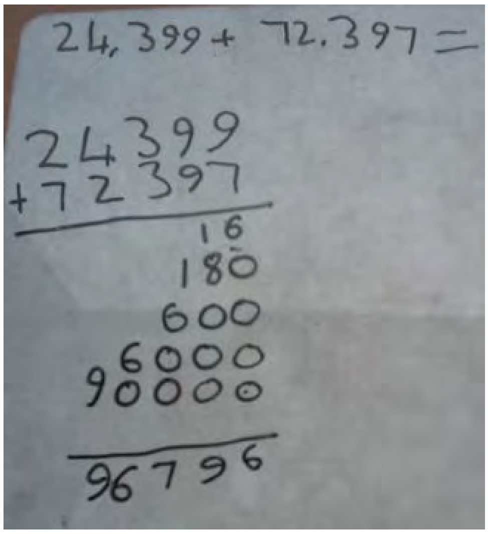




Find the total of the ones, then the tens, and finally the hundreds.

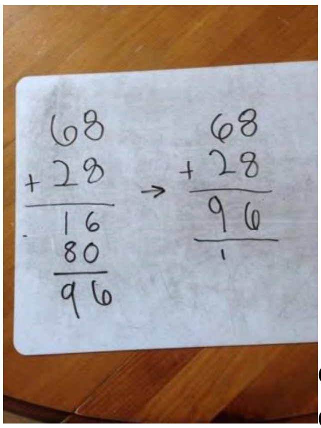


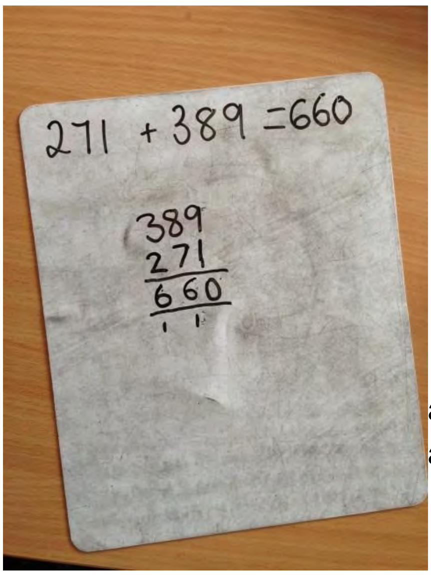
Lastly, find total of the rows.



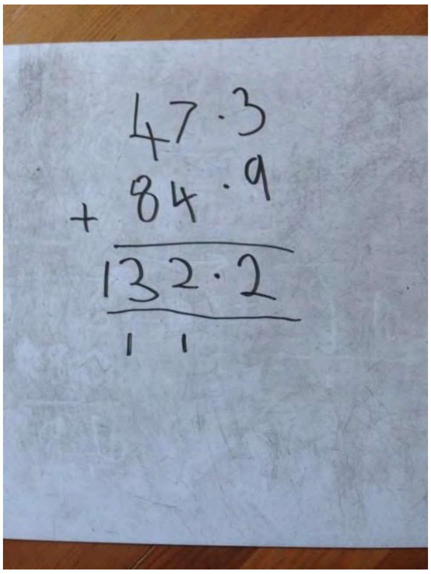
Compacted column addition

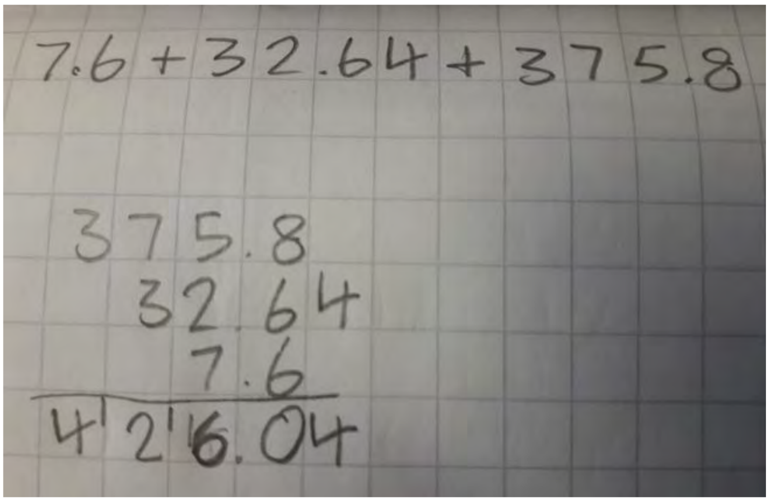
This method would not normally be used in lower school and even in years 5 and 6, there is no hurry to move to this. It is more important that children confidently use a method which, for them, is more efficient and accurate and that they still use the correct vocabulary of partitioning the 68 into 60 and 8 and the 28 into 20 and 8, adding the 6 and 8 to make 14, writing the 4 in the ones column and the 10 under the tens place before adding the tens together.



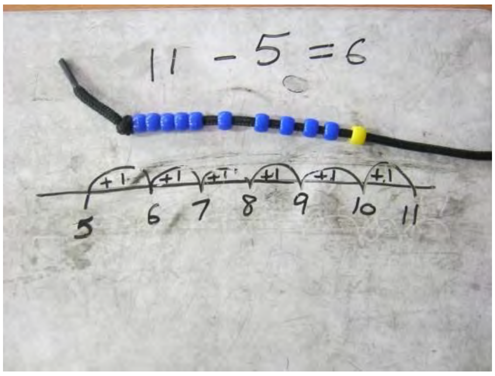


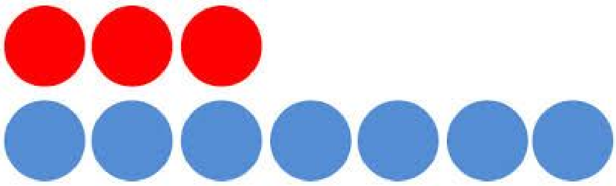
When confident and consistently accurate, pupils can then use either the expanded or compact method with larger numbers or decimals.





Subtraction





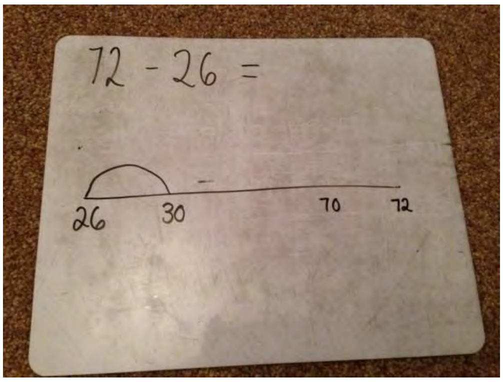
Finding the **difference**

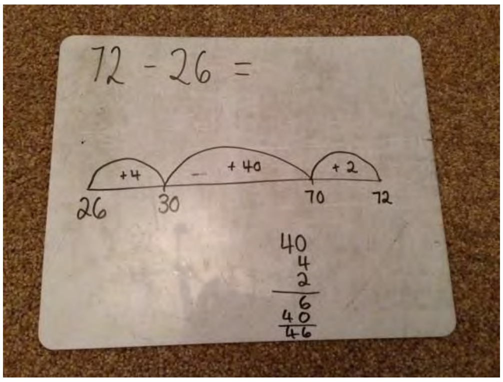
When subtracting, children need to understand that partitioning is not always appropriate.

Looking at the example of 73 – 26 = it is possible to start with 70 – 20 but
3 – 6 is less useful.

Number lines, however, make the calculation easy. This supports mental calculation and is a way of jotting down thoughts.

73 – 26 =





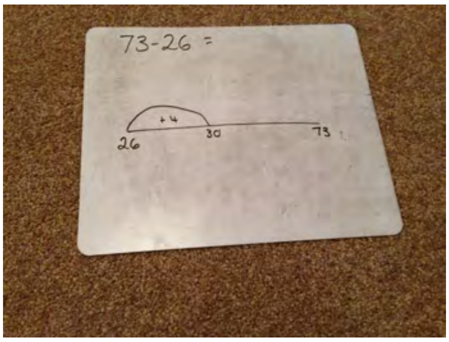
Counting up

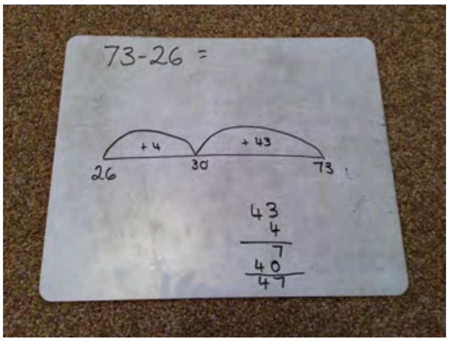
Begin this by drawing a line and starting it at the number which will be taken away. It is useful to mark the key numbers on the line first and then work out how big your jump needs to be.

To make it simpler, your first jump should be to the next multiple of 10. That then allows you to make a larger jump, in multiples of 10, to the multiple of 10 before the target number.

This use of number lines builds on the understanding of subtraction as difference or as complementary addition.

When they are confident with this stage, pupils can reduce the number of jumps.

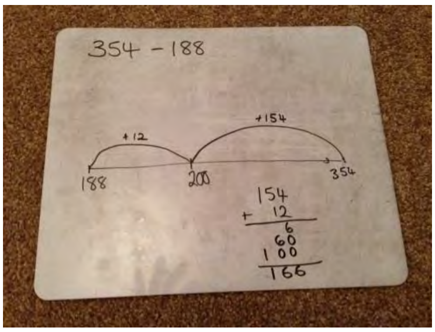




This method can be used for larger numbers by using complements to 100.

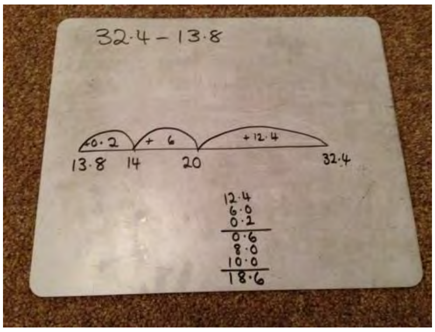
For most children, when they are ready to calculate with 3-digit numbers, they will move onto the expanded column method.

354 - 188



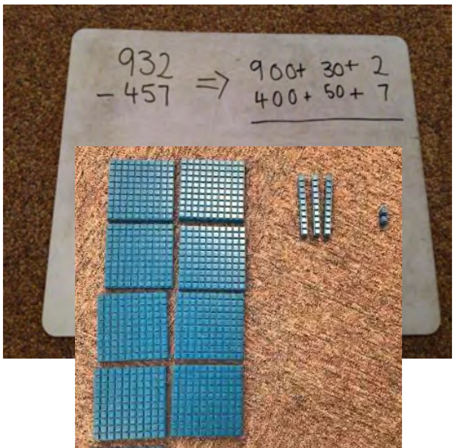
Subtraction of decimals is just as simple using the

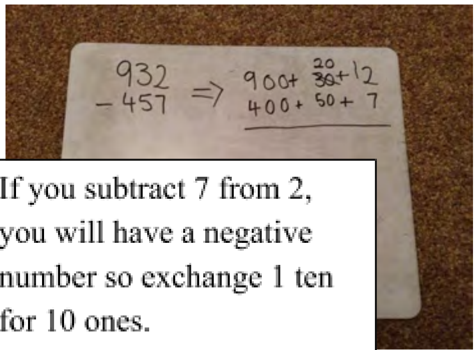
number line. **32.4 – 13.8**



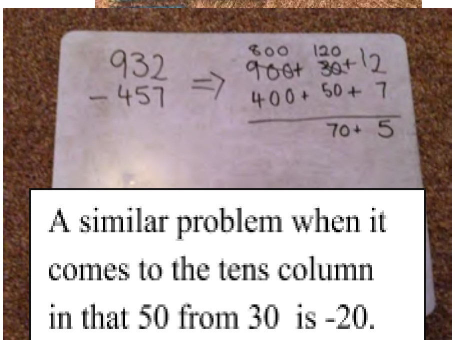
Expanded subtraction

Early vertical subtraction can begin with partitioning.

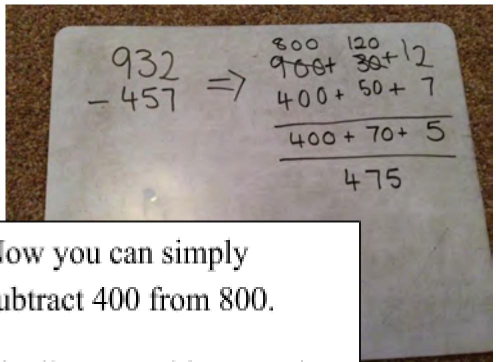




If you subtract 7 from 2, you will have a negative number so exchange 1 ten for 10 ones.

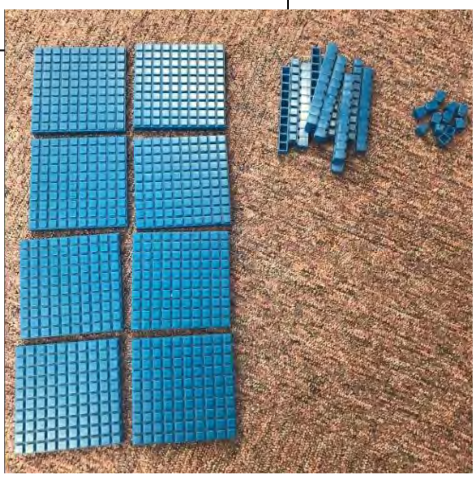


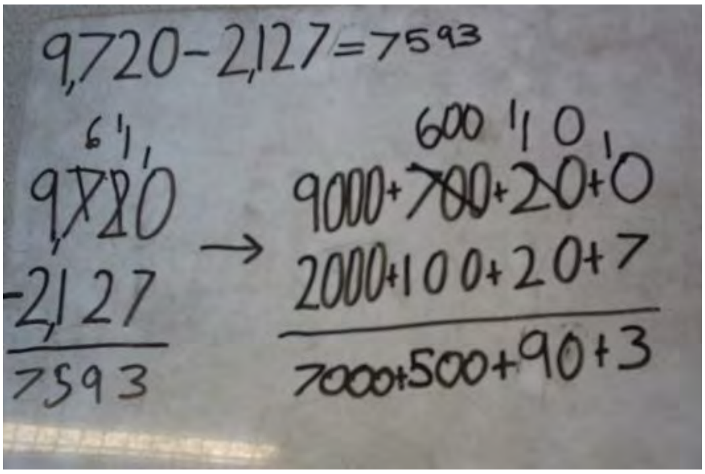
A similar problem when it comes to the tens column in that 50 from 30 is -20. Therefore, you need to exchange 1 hundred for 10 tens.



Now you can simply subtract 400 from 800.

Finally, recombine together the results of each subtraction to reach your answer.

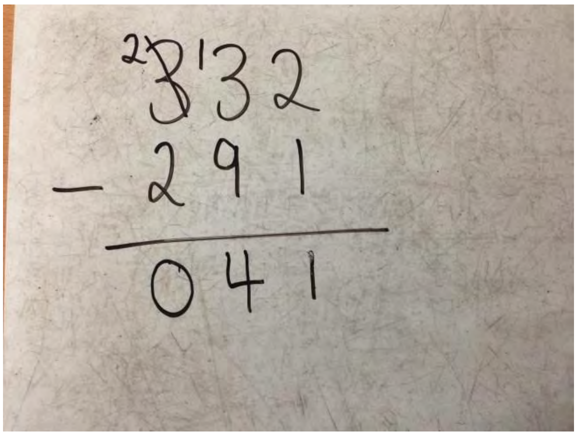




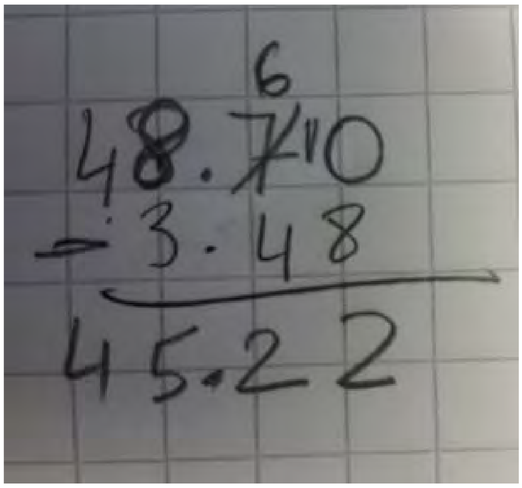
This is still suitable for much larger numbers if this is the conceptual stage of the child.

Compact subtraction

As with expanded subtraction, it will be necessary to exchange when it is not possible to subtract a number.



Instead of recording the 1's, 10's and 100's on separate lines, you must begin by subtracting 1's and recording this under the 1's and then the same with the 10's and 100's.



Multiplication

Times Table game

Arrays

Concepts of multiplication begin with doubling and are extended using the array**.** **Arrays** and **number lines** and **number sentences** will be the initial methods of recording.

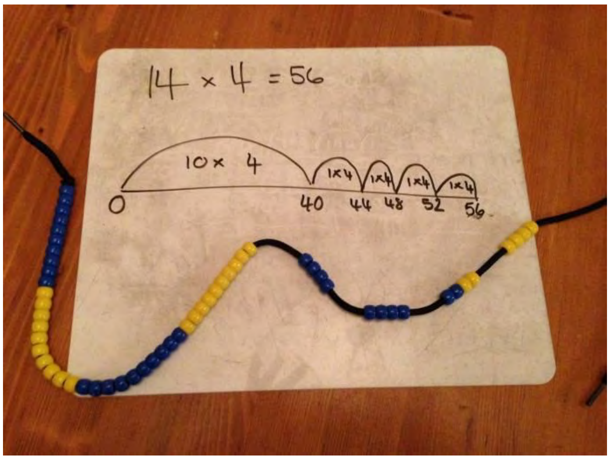
When pupils begin to multiply one-digit by two-digit numbers they will begin by using partitioning. They will need a good understanding of what they are doing. This will come from their work on arrays.

14 x 4

Partition this to…

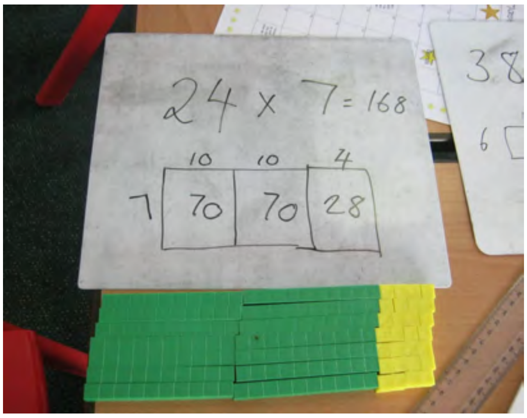
10 x 4 and 4 x 4.

The children could use a bead string to make 10 x 4 and then to make 4 groups of 4. This can then be recorded using a **numberline.**



This can then develop into the grid method.

24 x 7



24 x 7

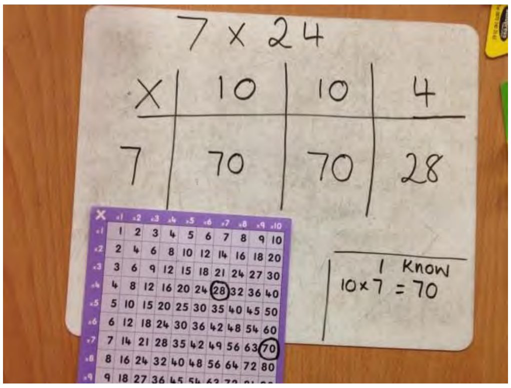
10 x 7 + 10 x 7 + 4 x 8

I know that 10 x 7 is the same as 7 x 10. I can use base 10 blocks to show that 10 x 7 =70.

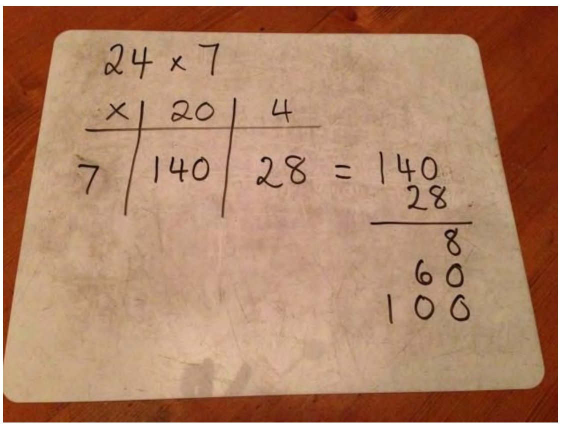
So, **20 x 7 = 140**

4 x 7 = 28

140 + 28 = 168



Leading to this… **Grid method**



7 x 24

10 x 7 + 10 x 7 + 4 x 8

I can use a multiplication grid to find 10 x 7.
I can then use it to find 4 x 7.

So, **20 x 7 = 140**

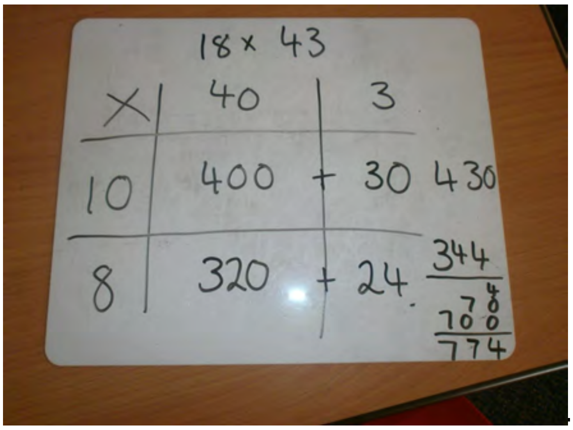
4 x 7 = 28

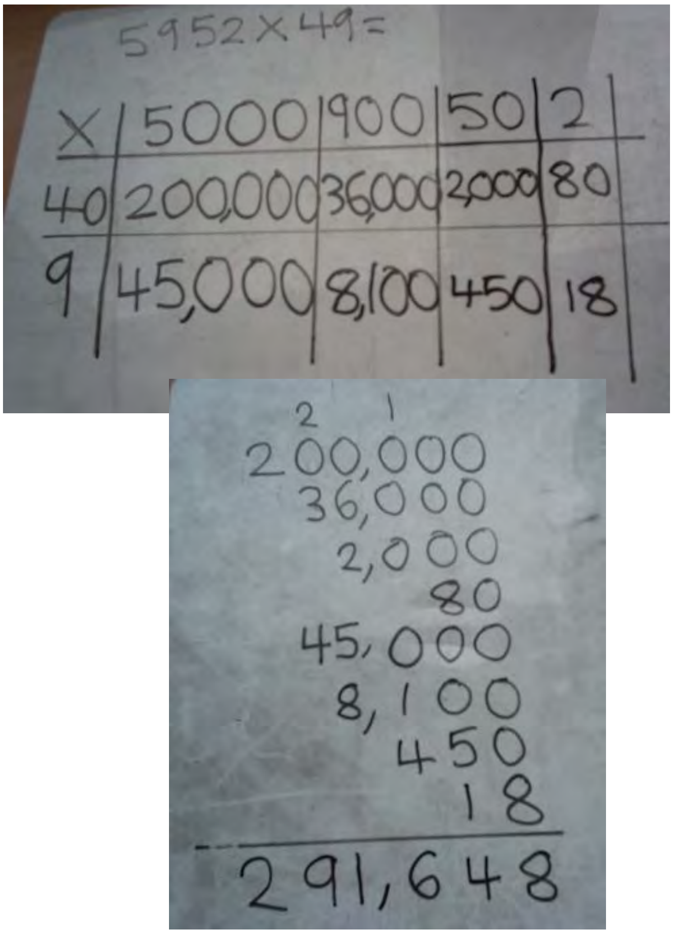
140 + 28 = 168

Grid method

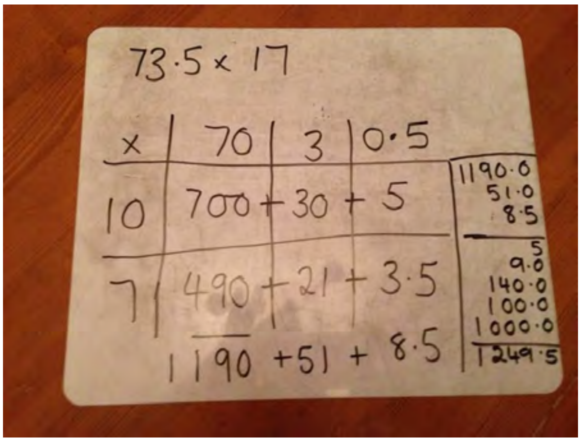
Once the children are confident and consistently accurate in multiplying one-digit by two-digit numbers using the grid method, they can also use the **grid method** for two-digit by two-digit multiplication.

18 x 43



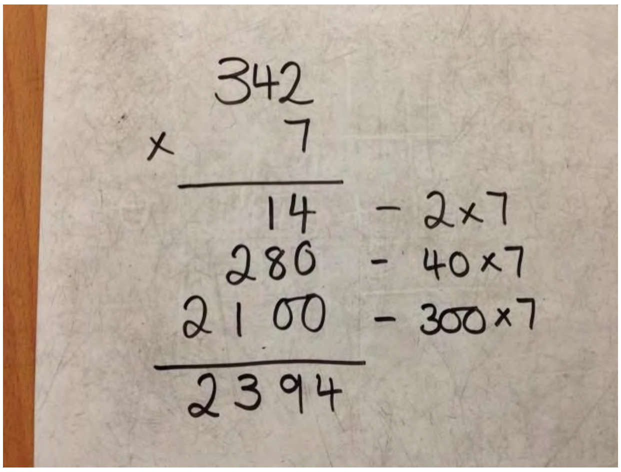


73.5 x 17



Expanded multiplication

Generally speaking, expanded multiplication should not be introduced to children until year 6 and should only be introduced then when they are confident in using grid method in a range of situations and have a secure understanding of place value. This should be taught alongside the grid initially so the children can see the origins of the partial products.



Partition the number that you are multiplying.

342 becomes **300 40** and **2**.

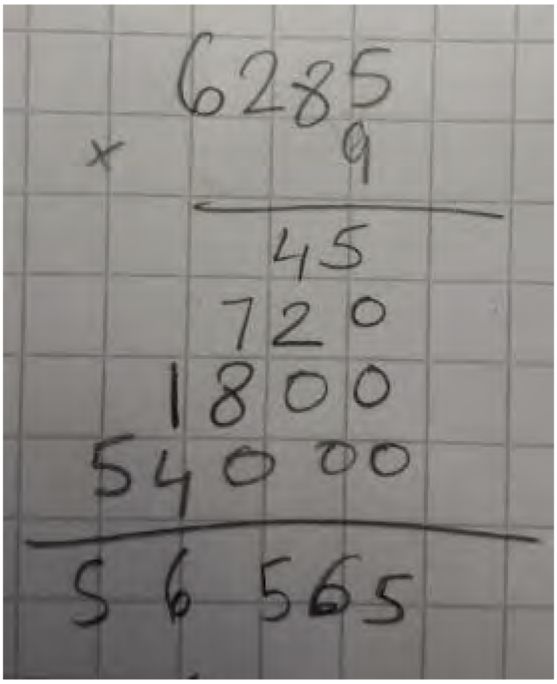
Record the totals of

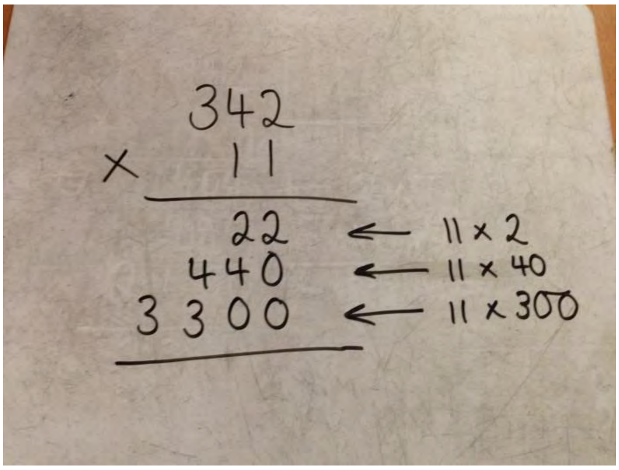
2 x 7

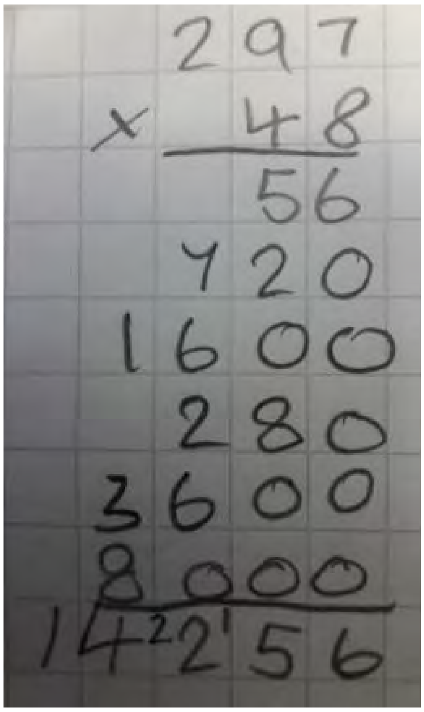
40 x 7

300 x 7

And then add them together.

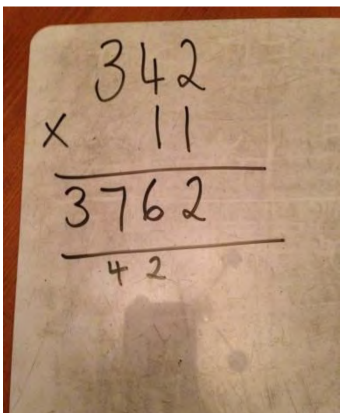






Compact multiplication

Instead of recording the ones, tens and hundreds multiplication on separate lines; the ones would be recorded in the ones column and the tens under the tens place and then added to the tens.



Division

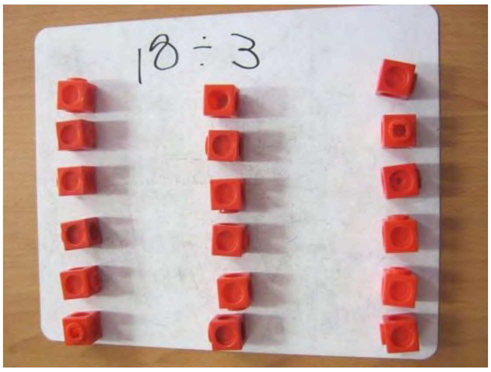
As with multiplication, division is recorded as **arrays, number lines** or **number sentences.**

18 ÷ 3

To begin with, division can be looked at as either **sharing** equally or **grouping** equally.

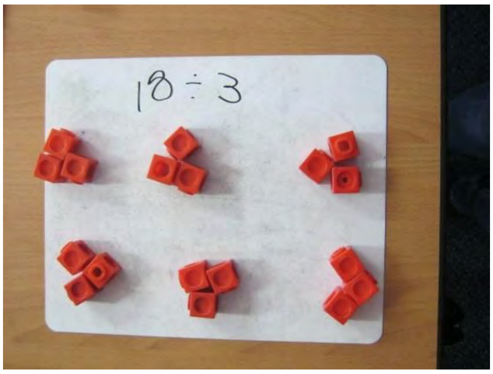
Sharing

Use counters, cubes or bears to **share** the given number equally into three groups. The amount of cubes in each group would be the answer.



Grouping

Use the counters and group them into three's. The amount of groups would be the answer.

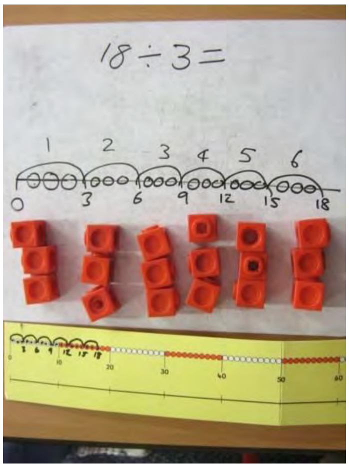


Using resources to physically explore grouping and sharing can help children understand they can both be used in early division.

Division on a number line.

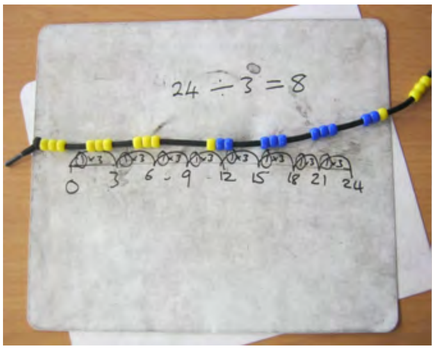
18 ÷ 3

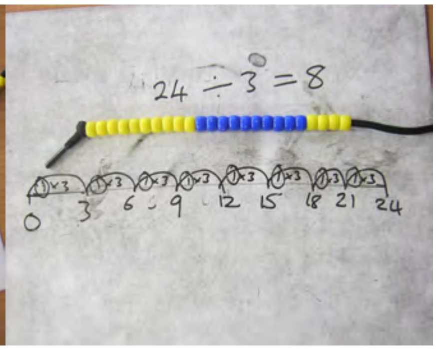
Once the children have a good understanding of grouping and sharing, they can then use this on a number line. Initially, it can be useful to use cubes to show how many are in each group or jump.

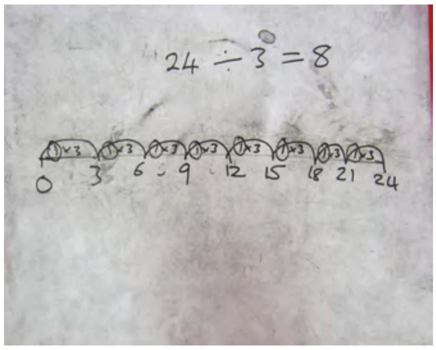


24 ÷ 3

I start at zero and count in 3's until I get to 24.



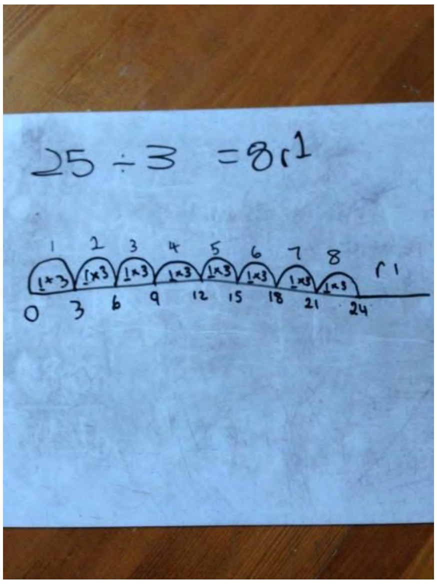




Bead strings can be used to share out the total number. Each group can represent one jump on the number line.

Calculations with remainders in the quotient can also be recorded on a number line.

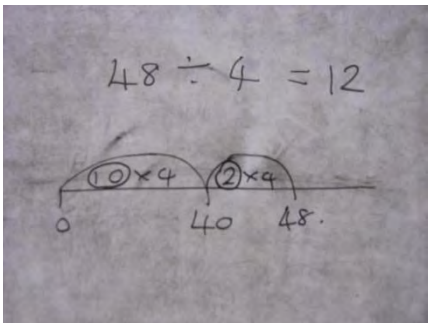
25 ÷ 3



I start at zero and count in 3's until I get to 24. Then there is 1 more to get to 25. So, I have 8 jumps of 3 and 1 remainder.

Number lines can also be used to work with more complex calculations, where the pupil is working with multiples of the divisor.

48 ÷ 4



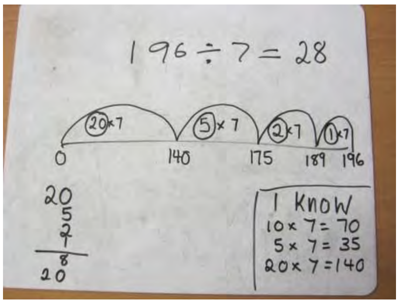
I must record the multiples of the divisor first when I record my jumps so that I know which numbers I must add together. I can also circle of underline this number to make it clear.

So **48 ÷ 4 = 12.**

196 ÷ 7

For many children, the addition of an 'I know' box can be very beneficial.

20 x 7 = 140
30 x 7 = 210 – this is too many so I will use 20 x 7 first.



A secure knowledge of place value is essential in order for the children to count on in multiples of the divisor.

When dividing decimals it is useful to begin by adapting a calculation that can already be understood.

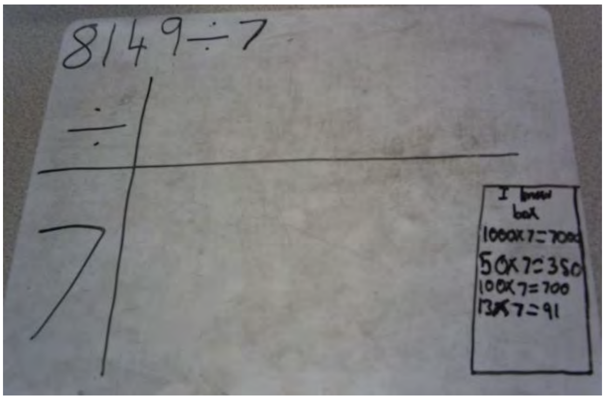
I know

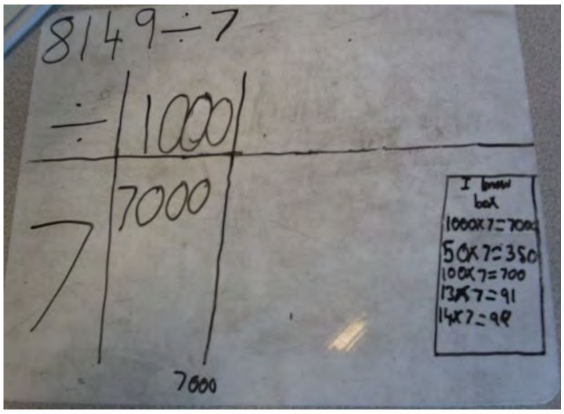
6 x 10 = 60	6 x 1 = 6	6 x 0.1 = 0.6
6 x 20 = 120	6 x 2 = 12	6 x 0.2 = 1.2
6 x 30 = 180	6 x 3 = 18	6 x 0.3 = 1.8

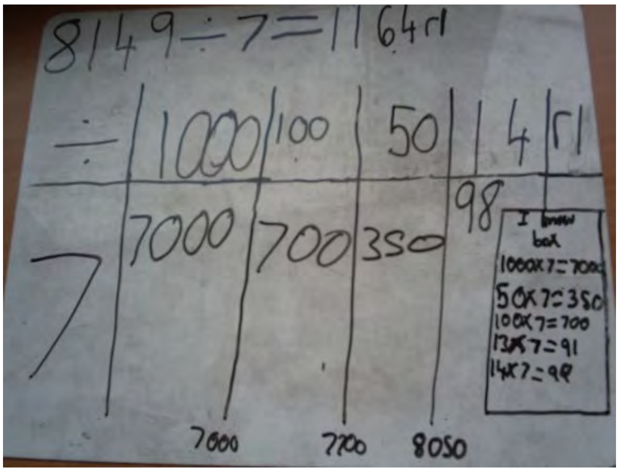
An ‘I know’ box is particularly important when dividing decimals.

Reverse Grid

When the children are secure using a number line to complete division calculations, the next step is to use the reverse grid method. This is something that is not usually attempted before year 4 and even then there is no hurry. It must only be attempted when the child is consistently accurate while using the number line.







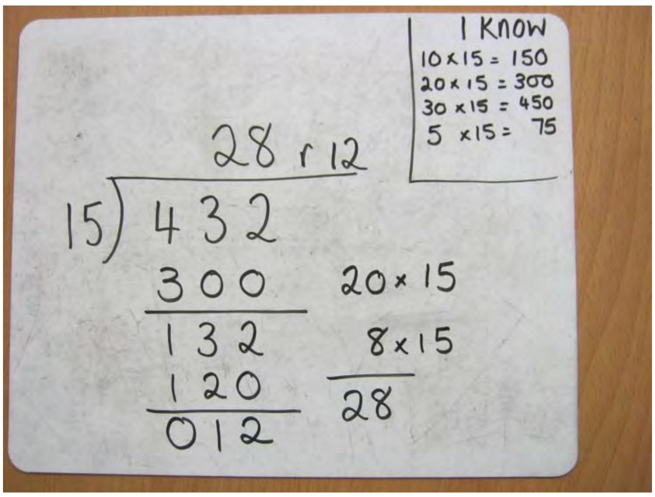
8149 **÷ 7 = 1164 1/7**

Children should continue to use an I know box. The idea is to try to keep the relationship with multiplication as clear as possible.

Vertical/Bus stop

The transition from the reverse grid to long division can be smooth if we look at long division as ‘vertical division’. The same methods of chunking can be used as in the number line method.

432 ÷ 15



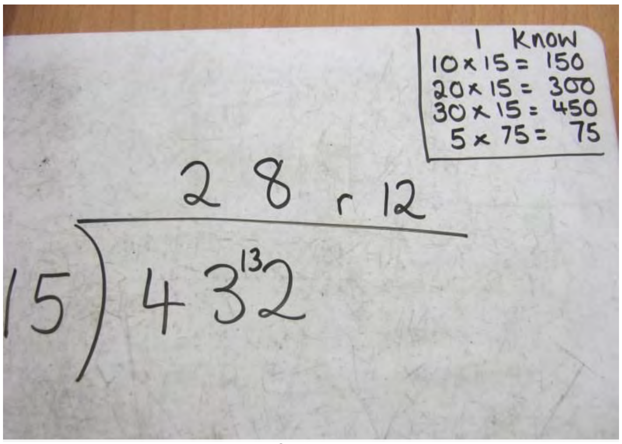
An ‘I know’ box can still be used if needed. If the child is secure in their number facts they may not need this and that is fine.

Using multiples of the divisor, the same as with number lines, to chunk larger sections of the total amount. Subtract these multiple amounts from the total. Continue until you do not have any more multiples of the divisor left. What is left is your **remainder.** Add up your multiples.

Once this method has been mastered, the children can move on to complete division calculations using **compact division.**

Compact division

This should not be attempted until year six and only then when it is apparent that the child is confident and has a good understanding of place value.



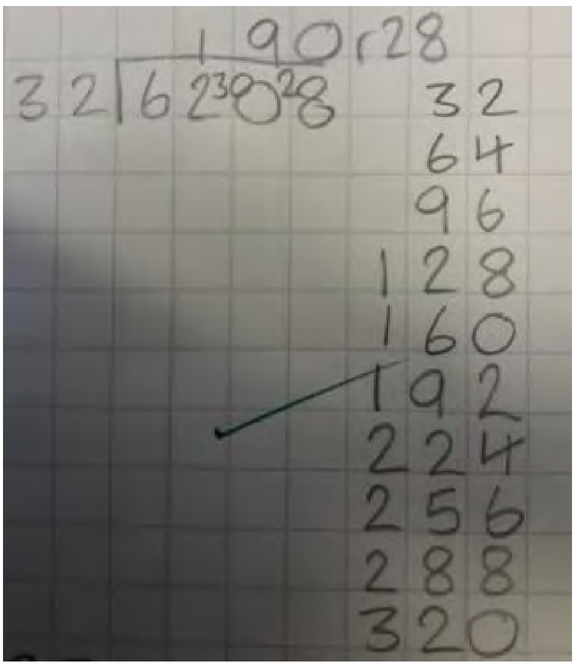
Use of an 'I know' box

15's into 43 = **2** with 13 remainder.

The remainder 13 is then placed in front of the 2 in the one's column. This now becomes 132. This is where the 'I know' box comes in.

'I know' that 10 x 15 = 150 – too many
'I know' that 9 x 15 = 135 – too many
'I know' that **8** x 15 = 120 – leaving **12 remainder**

12/15 = 4/5 = 0.8



6208 ÷ **32 = 190 r 28**

= 190 28/32

= 190 7/8 = 190.875