Dereham Church of England

Calculation Policy

Addition and Subtraction

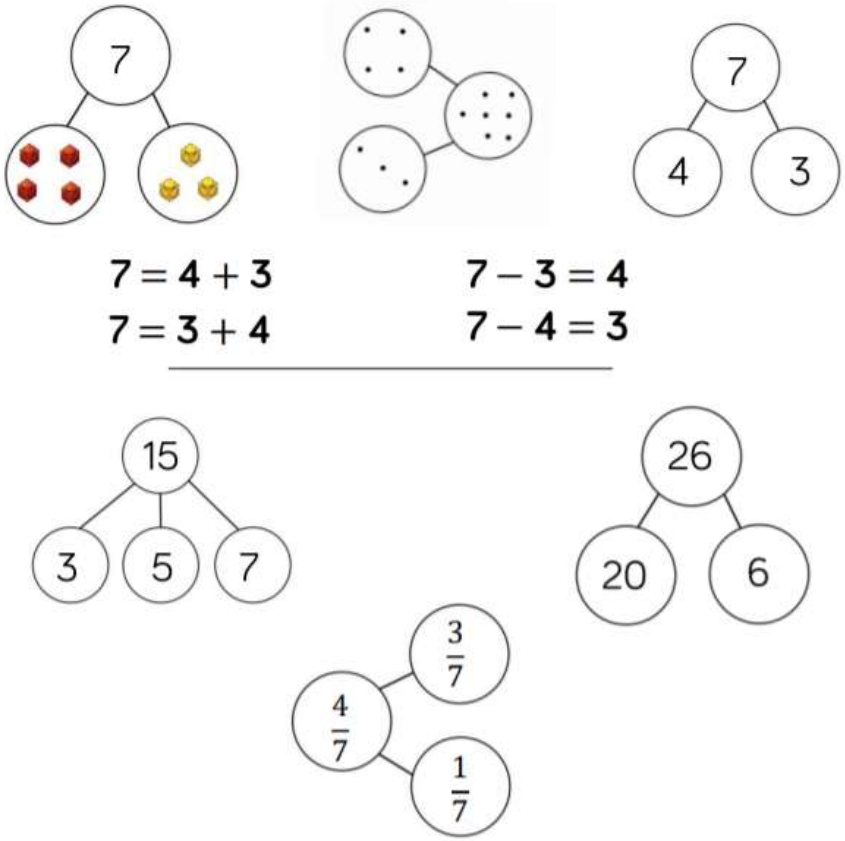
The full set of addition calculations that pupils need to be able to solve with automaticity are shown in the table below. Pupils must also be able to solve the corresponding subtraction calculations with automaticity.

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

Pupils must be fluent in these facts by the end of year 2, and should continue with regular practice through year 3 to secure and maintain fluency. It is essential that pupils have automatic recall of these facts before they learn the formal written methods of columnar addition and subtraction.

Models and Images

Part-Whole Model



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

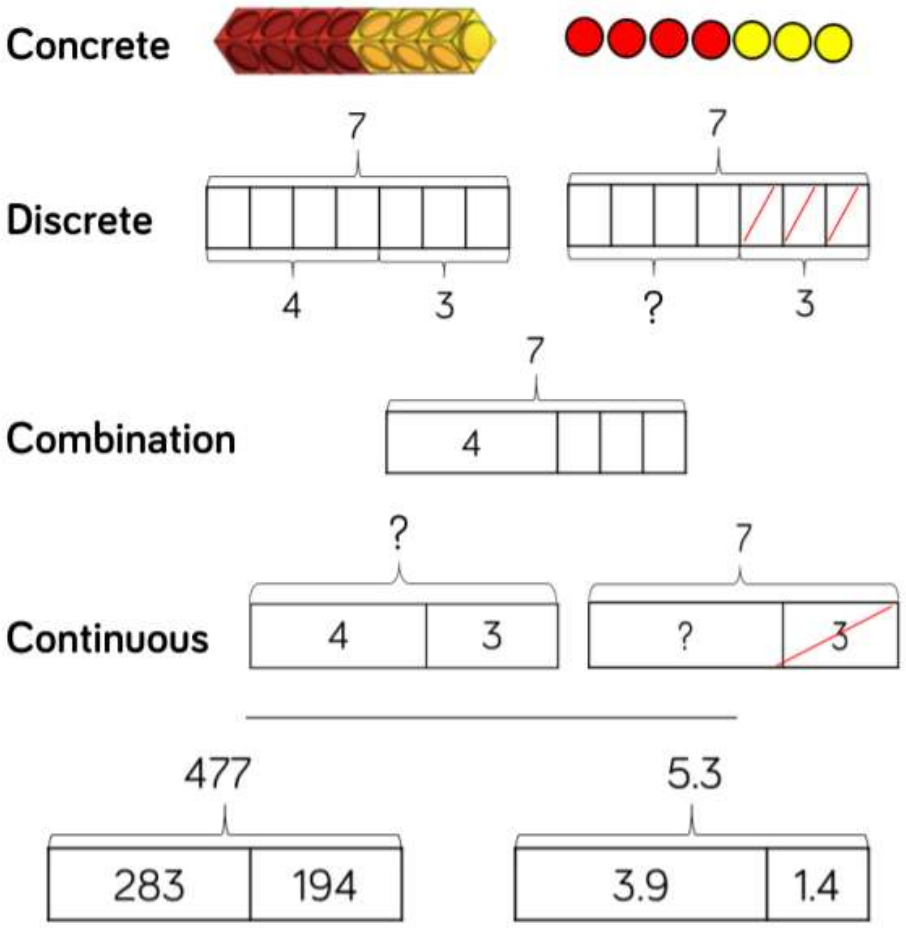
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

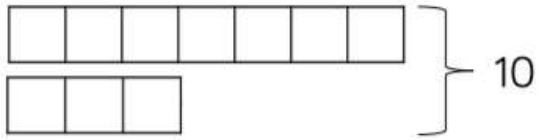
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

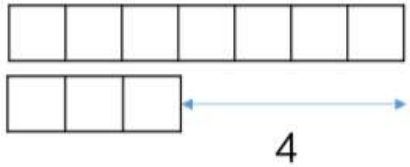
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

Discrete

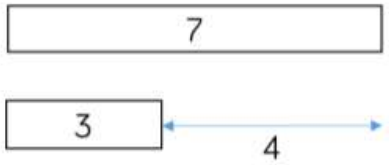


$7 + 3 = 10$

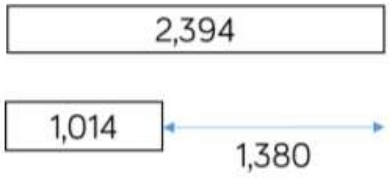


$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2{,}394 - 1{,}014 = 1{,}380$

Benefits

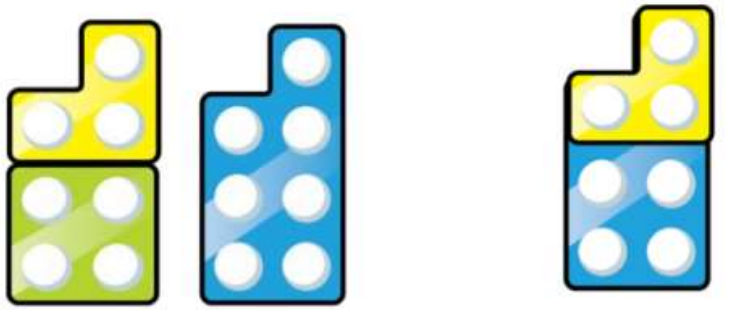
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

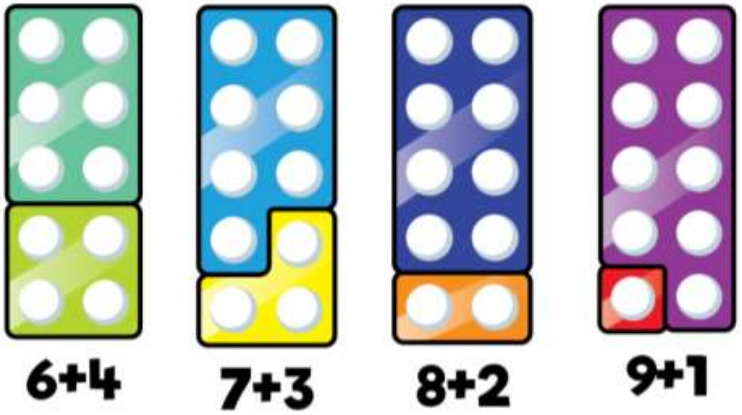
Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes



$7 = 4 + 3$ $7 = 3 + 4$ $7 - 3 = 4$



Benefits

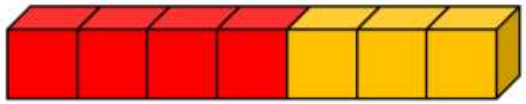
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

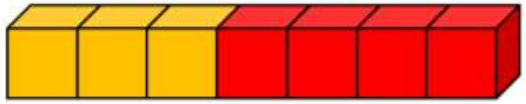
When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

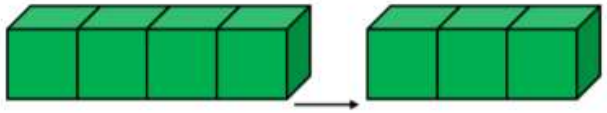
Cubes



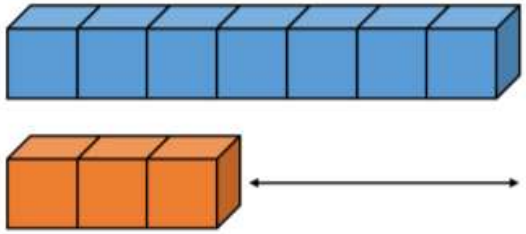
$7 = 4 + 3$



$7 = 3 + 4$



$7 - 3 = 4$



$7 - 3 = 4$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

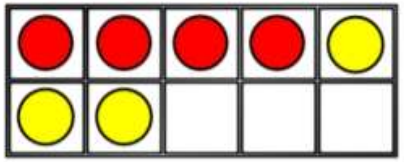
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

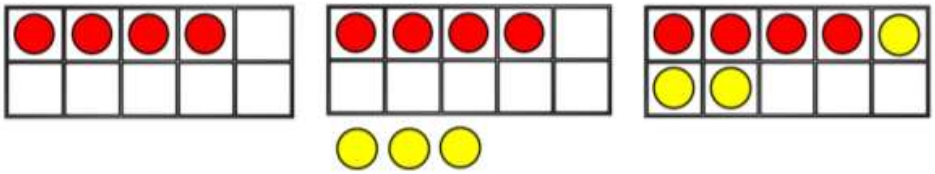
Ten Frames (within 10)



$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

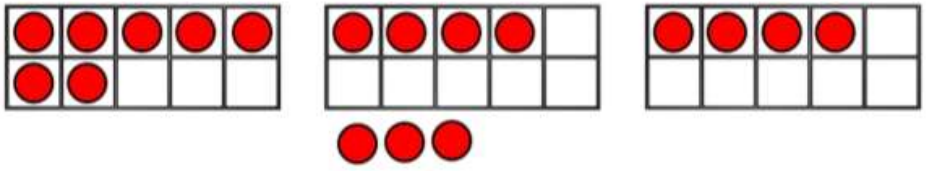
4 is a part.
3 is a part.
7 is the whole.

First | Then | Now



$4 + 3 = 7$

First | Then | Now



$7 - 3 = 4$

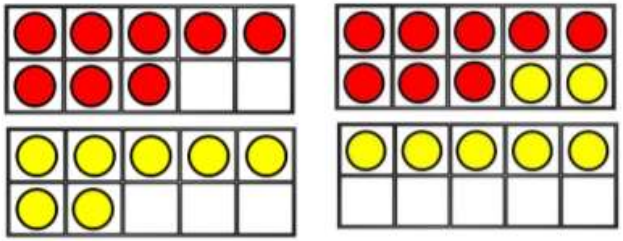
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

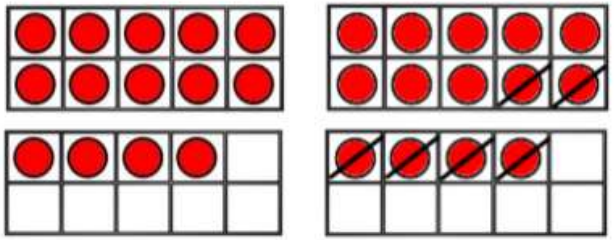
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



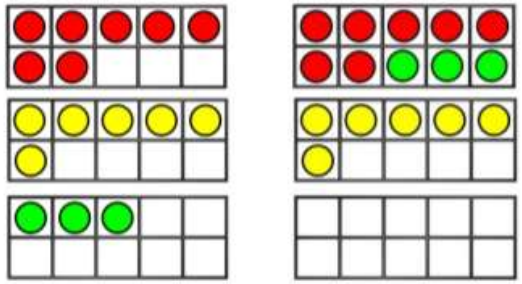
$8 + 7 = 15$

2 5



$14 - 6 = 8$

4 2



$7 + 6 + 3 = 16$

10

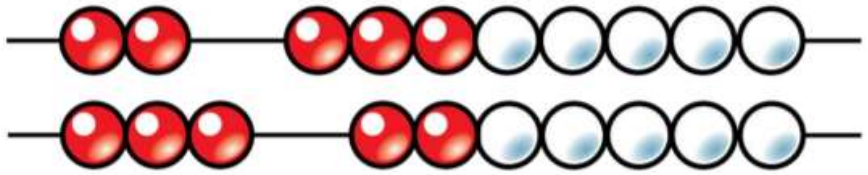
Benefits

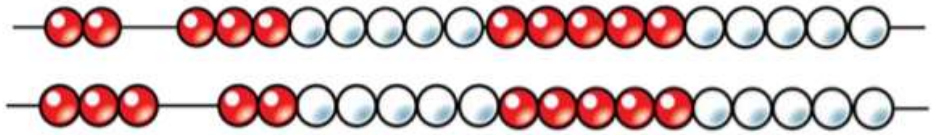
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

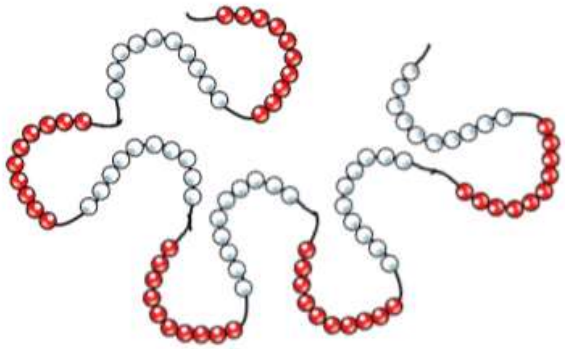
When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings







Benefits

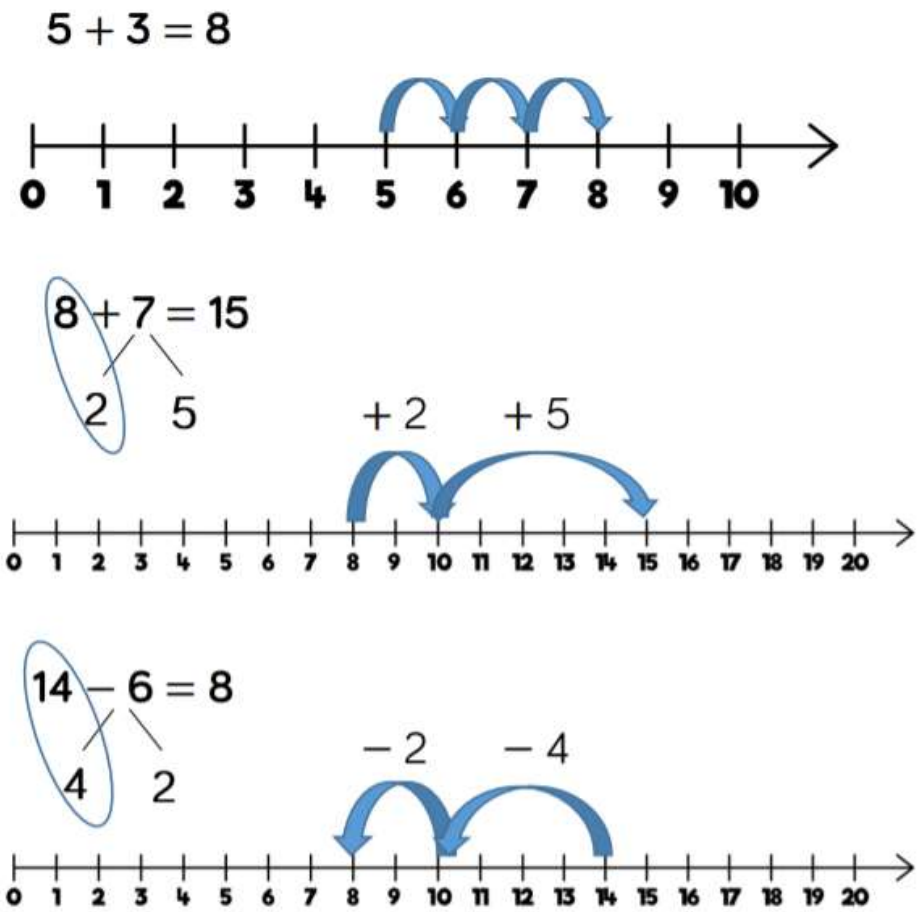
Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. 2 + 8 = 10, move one bead, 3 + 7 = 10.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Number Lines (labelled)



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

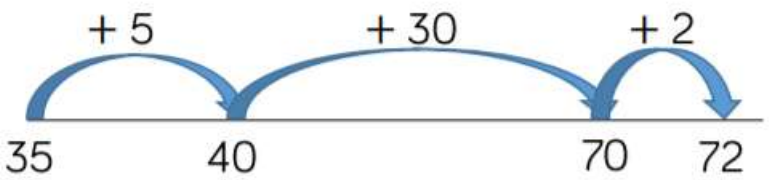
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

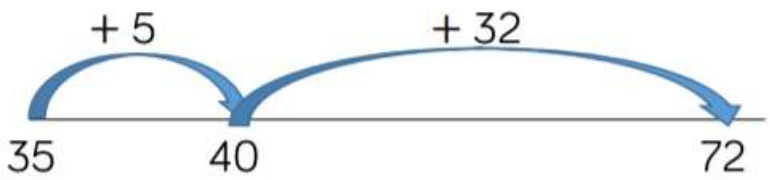
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

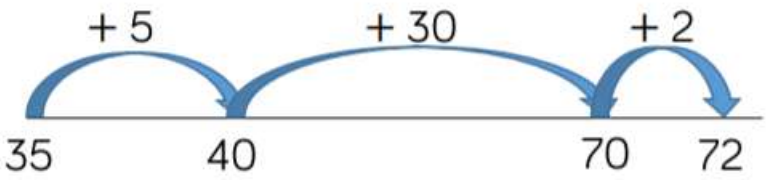
35 + 37 = 72



35 + 37 = 72



72 − 35 = 37



Benefits

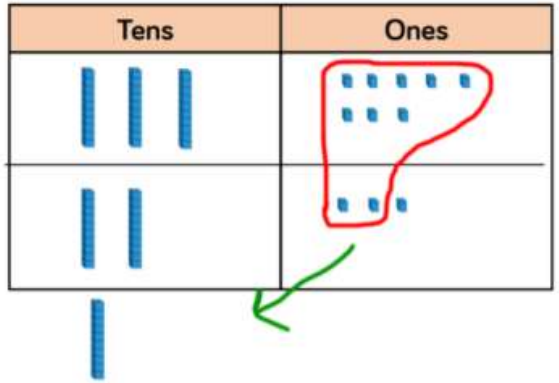
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

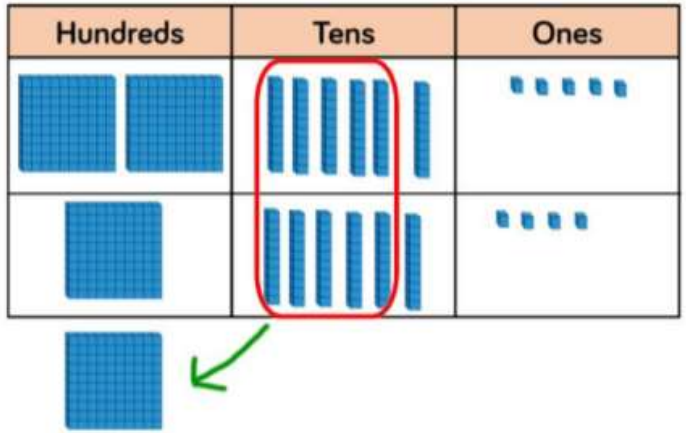
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$$

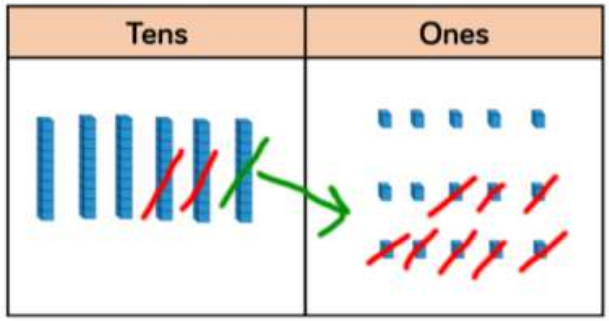
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

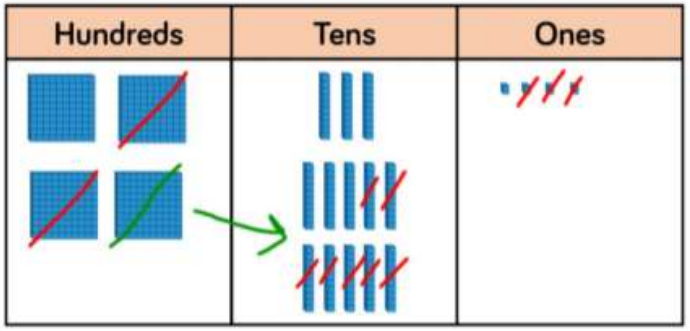
Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}5 \\ -\ 28 \\ \hline 37 \end{array}$$



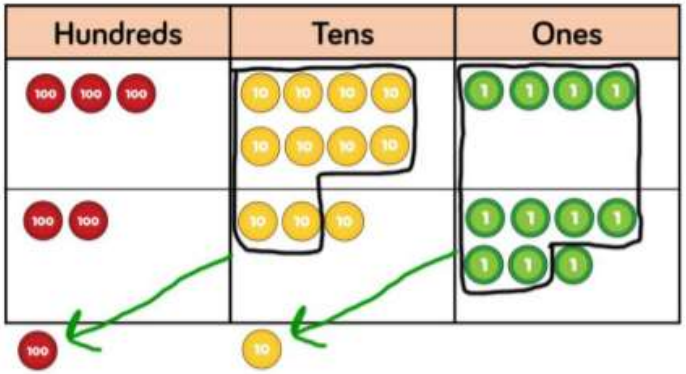
$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}35 \\ -\ 273 \\ \hline 262 \end{array}$$

Benefits

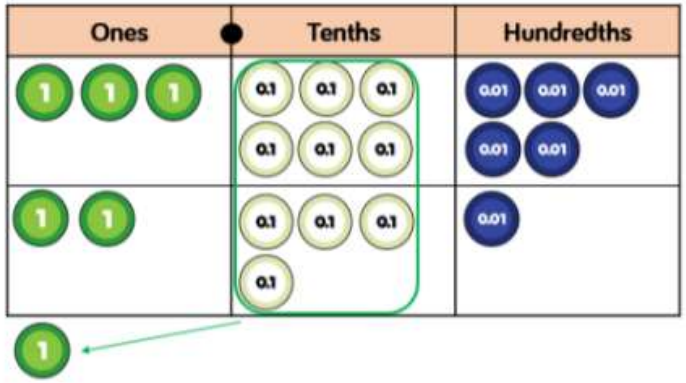
Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ \ 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

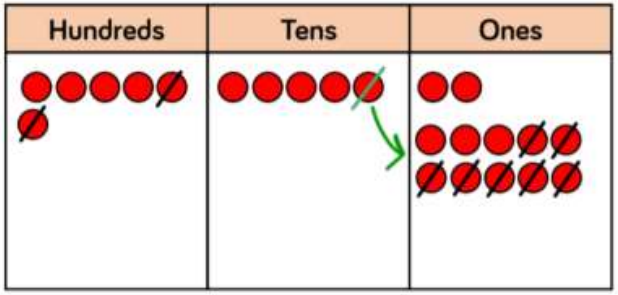
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

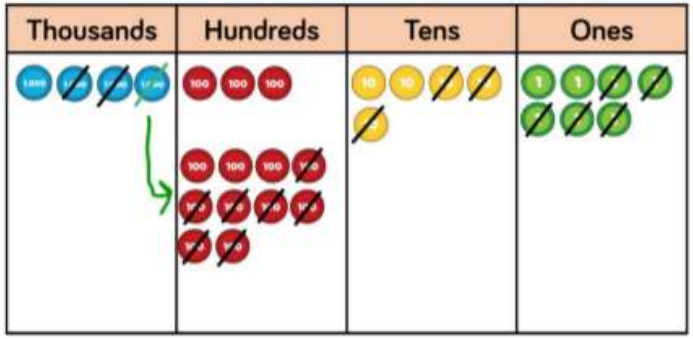
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)



$$\begin{array}{r} 6\cancel{5}^{4}\,{}^{1}2 \\ -\ 207 \\ \hline 445 \\ \hline \end{array}$$



$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}357 \\ -\ 2735 \\ \hline 1622 \\ \hline \end{array}$$

Benefits

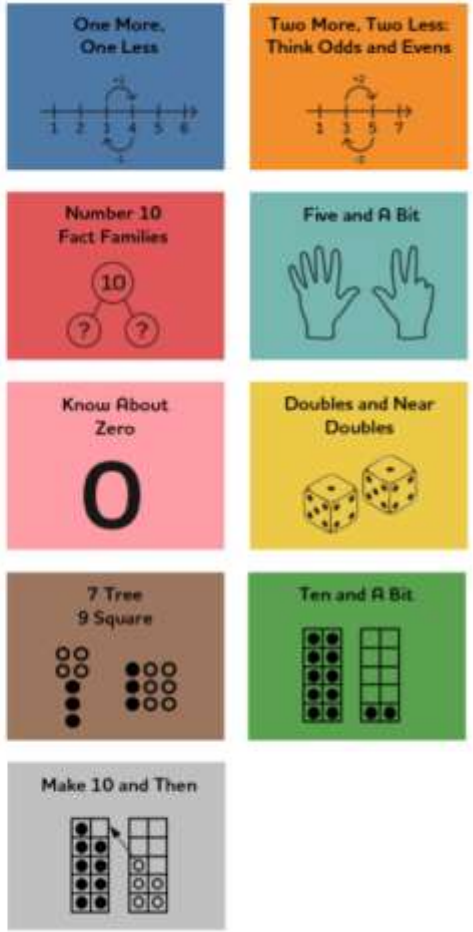
Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

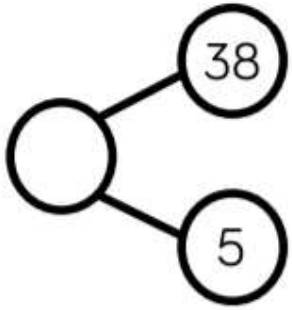
When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

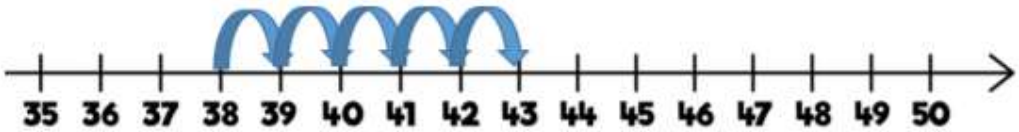
ADDITION

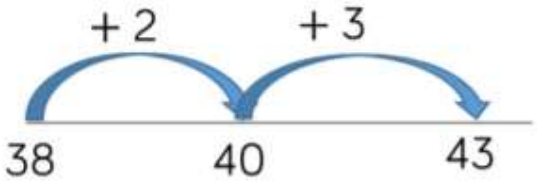
+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

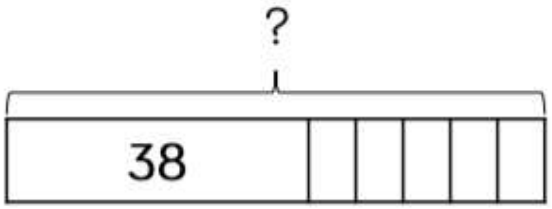


Skill: Add 1-digit and 2-digit numbers to 100

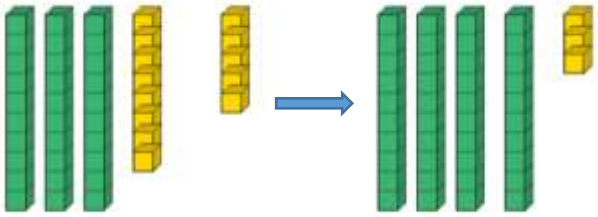






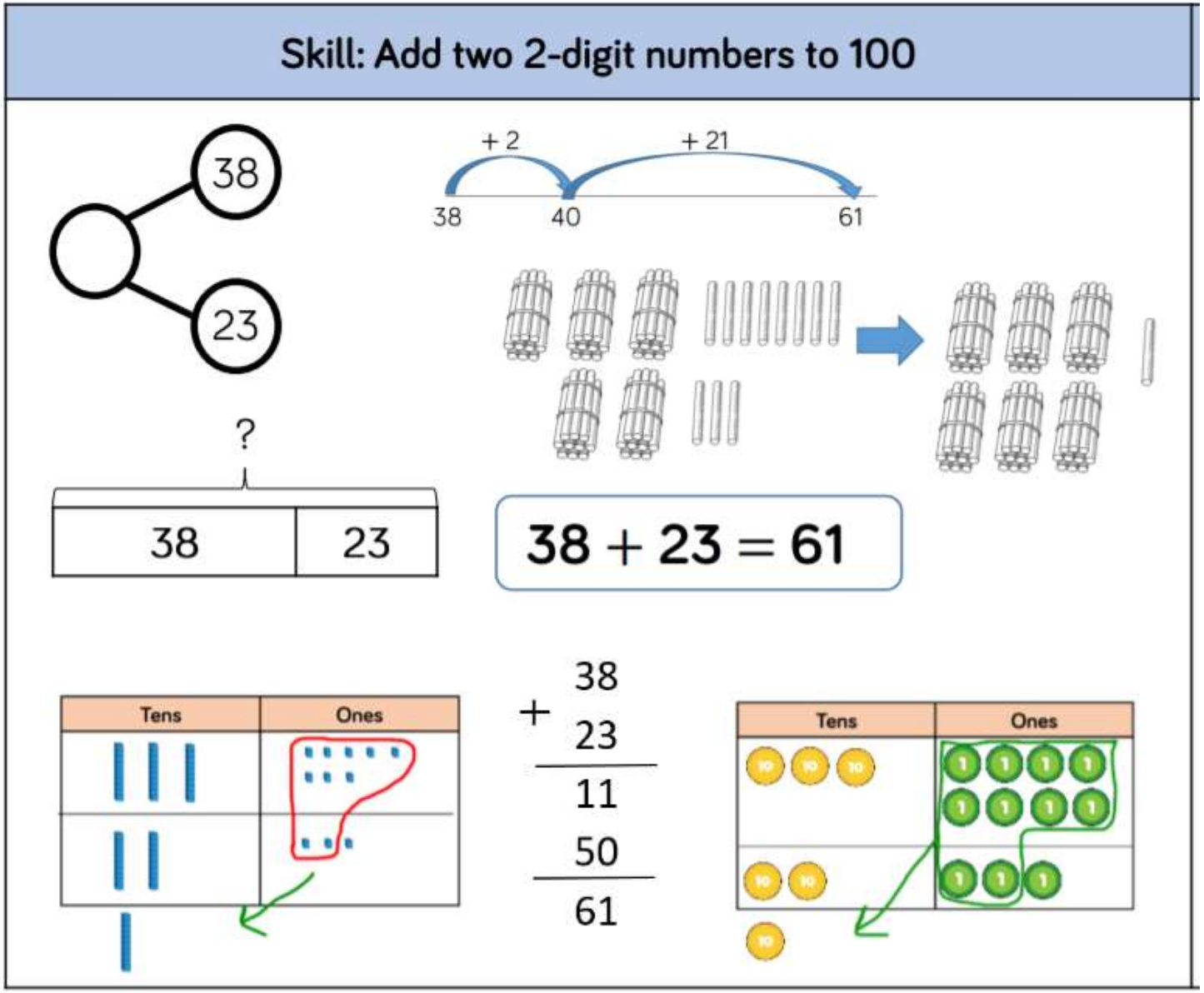


$$38 + 5 = 43$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

When adding a single digit to a two-digit number, children should be encouraged to calculate from the larger number by applying their **knowledge of number bonds.**



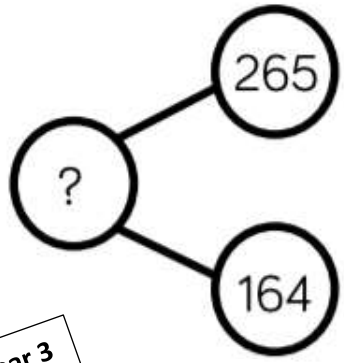
Encourage children to use a formal column method alongside other manipulatives. These can be concrete and then pictorial depending on the understanding of the child.

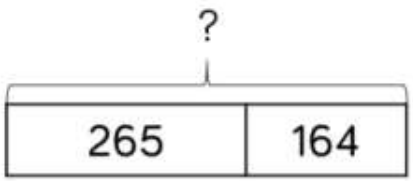
When understanding is secure, manipulatives can be withdrawn.

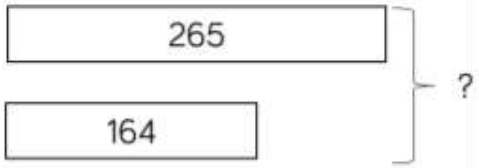
8 ones add 3 ones equals 11 ones.

3 tens add 2 tens equals 5 tens

Skill: Add numbers with up to 3 digits







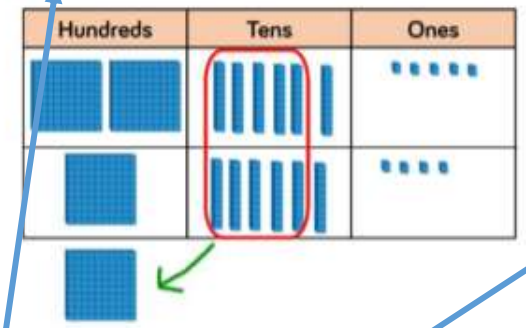
Year 3

+ 200 + 60 + 5
100 + 60 + 4

400 + 20 + 9

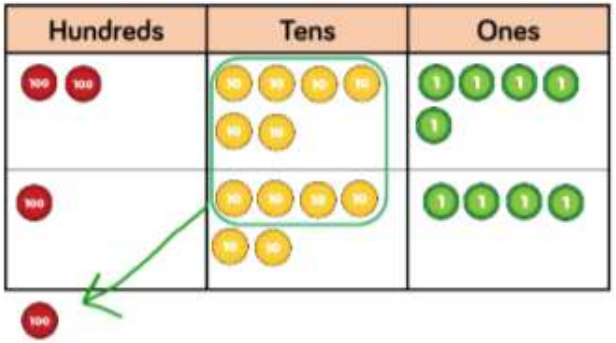
100

$$265 + 164 = 429$$



Year 4

265
+ 164
429
1



Below the line

Language focus

"8 plus 6 is equal to 14, so 8 hundreds plus 6 hundreds is equal to 14 hundreds."

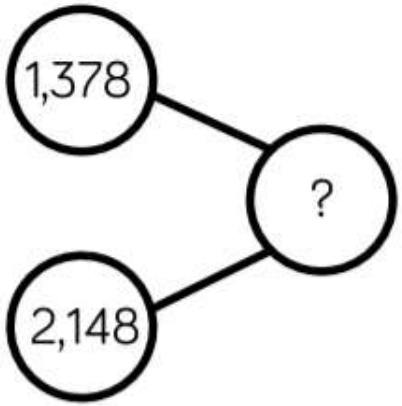
"14 hundreds is equal to 1,400."

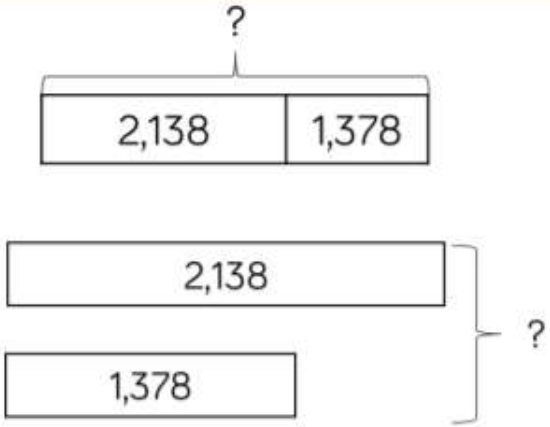
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

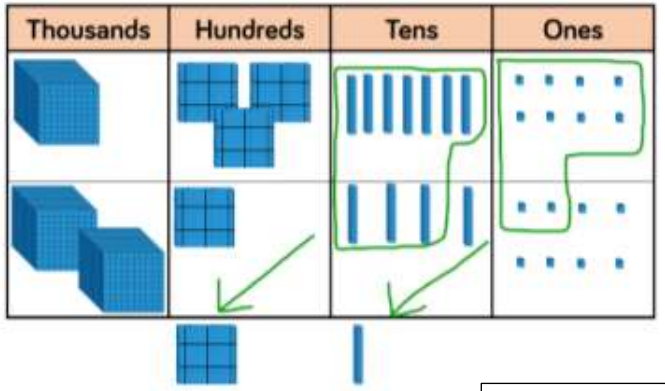
Skill: Add numbers with up to 4 digits

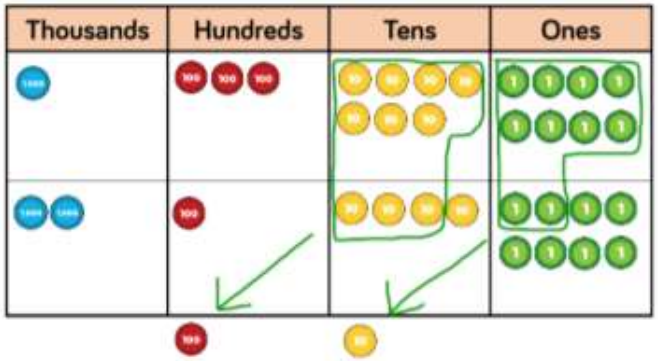




	1	3	7	8
+	2	1	4	8
	3	5	2	6
		1	1	

1,378 + 2,148 = 3,526





Language focus

"8 plus 6 is equal to 14, so 8 hundreds plus 6 hundreds is equal to 14 hundreds."

"14 hundreds is equal to 1,400."

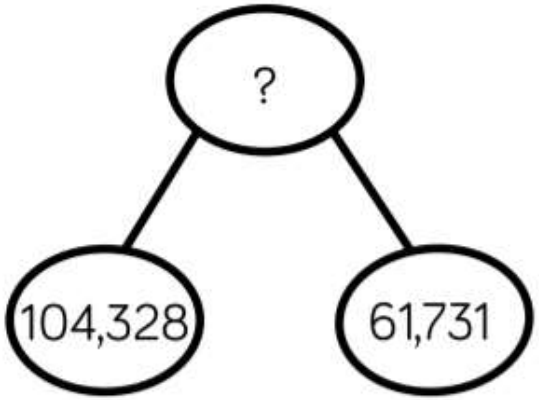
Below the line

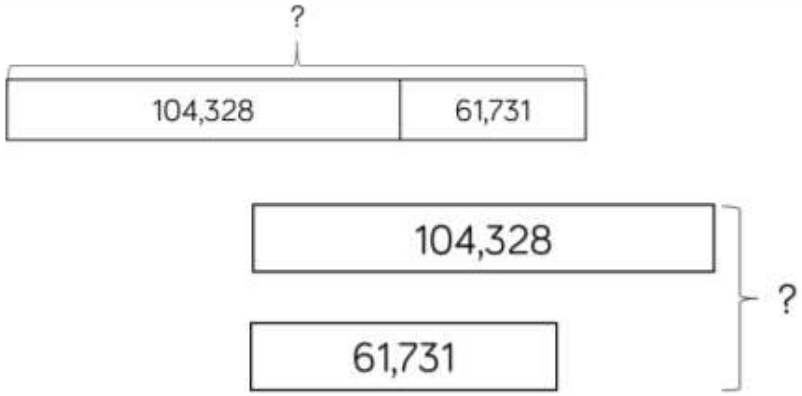
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

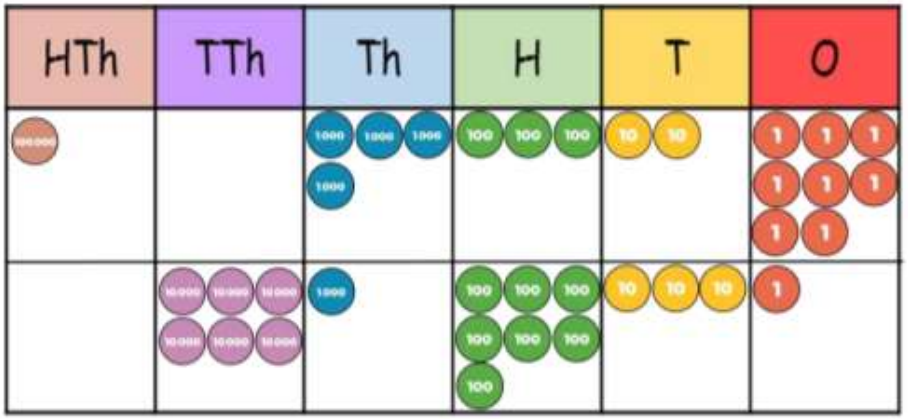
Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with more than 4 digits





104,328 + 61,731 = 166,059



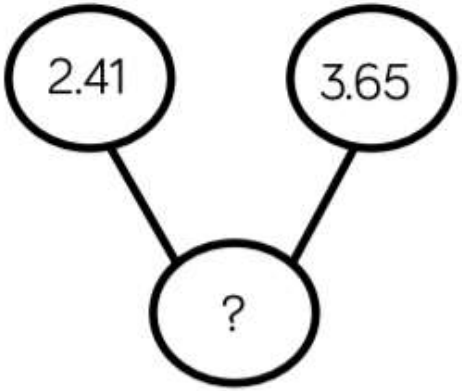
1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
		1			

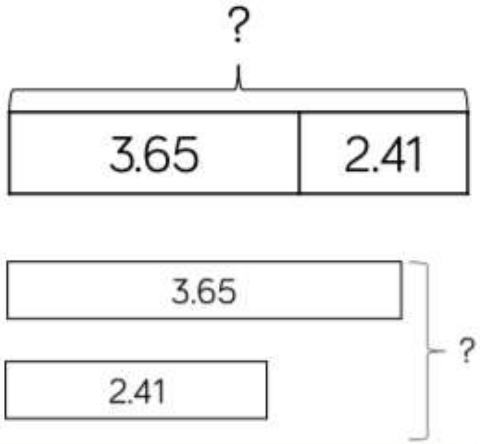
Below the line

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

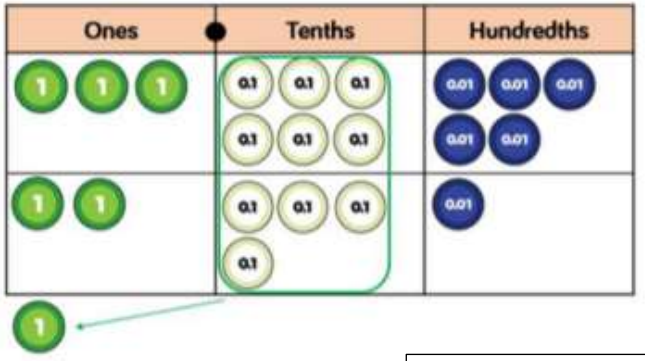
Skill: Add with up to 3 decimal places

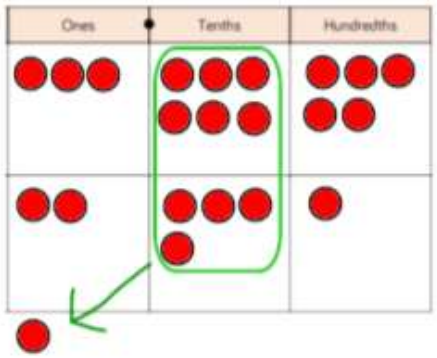




$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

3.65 + 2.41 = 6.06





Language focus

"8 plus 6 is equal to 14, so 8 hundreds plus 6 hundreds is equal to 14 hundreds."

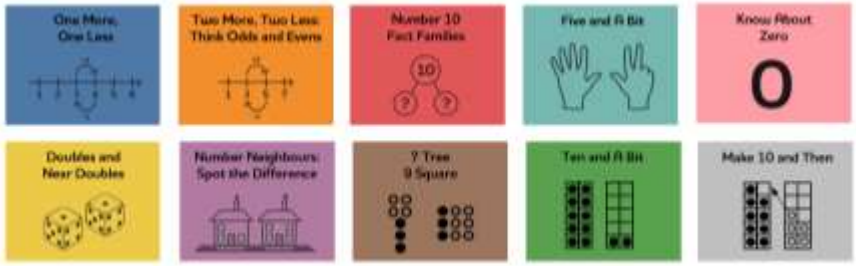
"14 hundreds is equal to 1,400."

Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

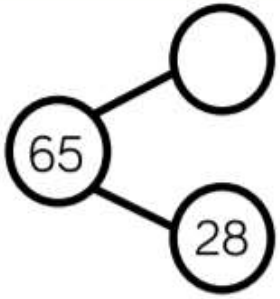
Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

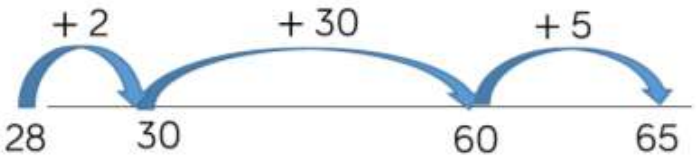
SUBTRACTION

–	0	1	2	3	4	5	6	7	8	9	10
0	0 – 0										
1	1 – 0	1 – 1									
2	2 – 0	2 – 1	2 – 2								
3	3 – 0	3 – 1	3 – 2	3 – 3							
4	4 – 0	4 – 1	4 – 2	4 – 3	4 – 4						
5	5 – 0	5 – 1	5 – 2	5 – 3	5 – 4	5 – 5					
6	6 – 0	6 – 1	6 – 2	6 – 3	6 – 4	6 – 5	6 – 6				
7	7 – 0	7 – 1	7 – 2	7 – 3	7 – 4	7 – 5	7 – 6	7 – 7			
8	8 – 0	8 – 1	8 – 2	8 – 3	8 – 4	8 – 5	8 – 6	8 – 7	8 – 8		
9	9 – 0	9 – 1	9 – 2	9 – 3	9 – 4	9 – 5	9 – 6	9 – 7	9 – 8	9 – 9	
10	10 – 0	10 – 1	10 – 2	10 – 3	10 – 4	10 – 5	10 – 6	10 – 7	10 – 8	10 – 9	10 – 10
11		11 – 1	11 – 2	11 – 3	11 – 4	11 – 5	11 – 6	11 – 7	11 – 8	11 – 9	11 – 10
12			12 – 2	13 – 3	12 – 4	12 – 5	12 – 6	12 – 7	12 – 8	11 – 9	12 – 10
13				13 – 3	13 – 4	13 – 5	13 – 6	13 – 7	13 – 8	13 – 9	13 – 10
14					14 – 4	14 – 5	14 – 6	14 – 7	14 – 8	14 – 9	14 – 10
15						15 – 5	15 – 6	15 – 7	15 – 8	15 – 9	15 – 10
16							16 – 6	16 – 7	16 – 8	16 – 9	16 – 10
17								17 – 7	17 – 8	17 – 9	17 – 10
18									18 – 8	18 – 9	18 – 10
19										19 – 9	19 – 10
20											20 – 10

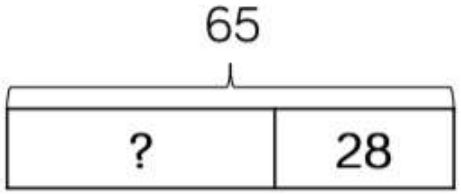


Skill: Subtract 1 and 2-digit numbers to 100

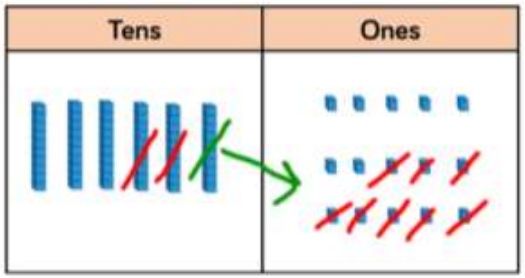


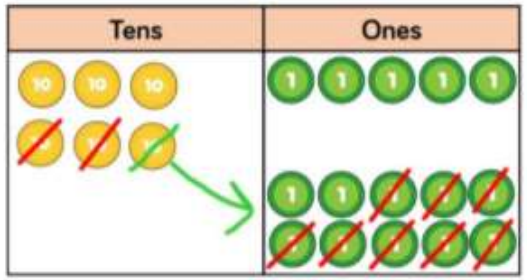


$$\begin{array}{r} \overset{50}{\cancel{60}} + {}^{1}5 \\ - \quad 20 + 8 \\ \hline 30 + 7 \end{array} = 37$$



$$65 - 28 = 37$$



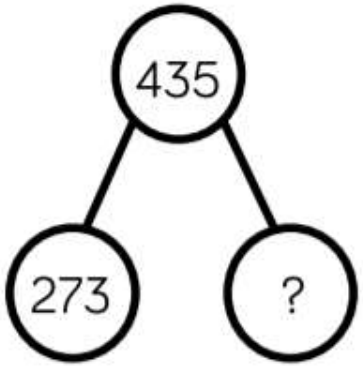


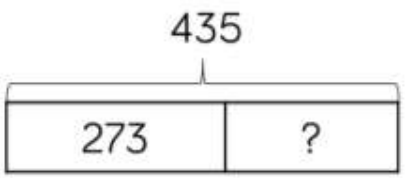
★ Again, encourage children to calculate using number bonds rather than counting in ones.

At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the difference.★ Encourage them to jump to multiples of 10 to become more efficient.

Skill: Subtract numbers with up to 3 digits

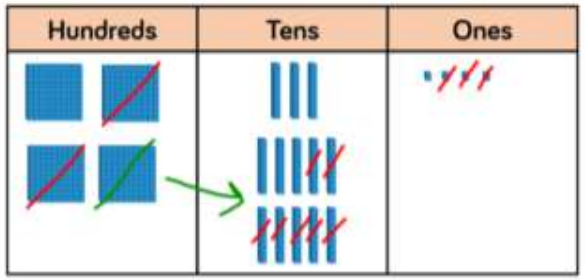


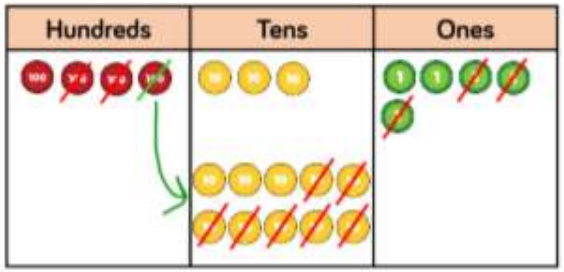


$$435 - 273 = 262$$

300

$$\begin{array}{r} \cancel{400} + {}^{1}30 + 5 \\ - \quad 200 + \ 70 + 3 \\ \hline 100 + 60 + 2 \\ \hline \end{array}$$





Language focus

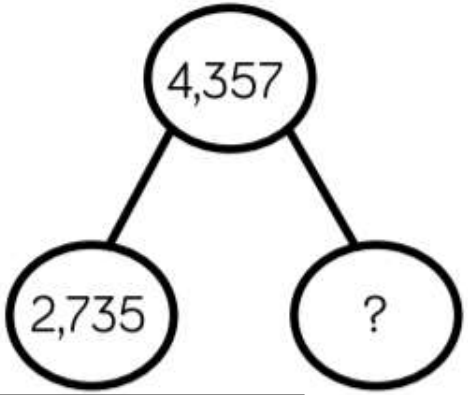
"5 ones minus 3 ones is equal to 2 ones."
"6 tens minus 2 tens is equal to 4 tens."

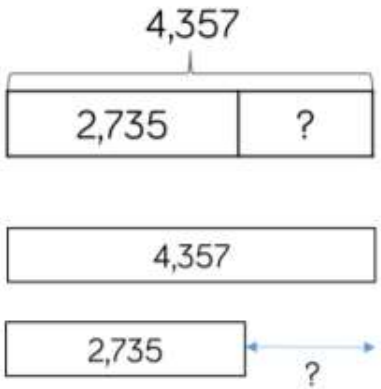
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with up to 4 digits





Year 5

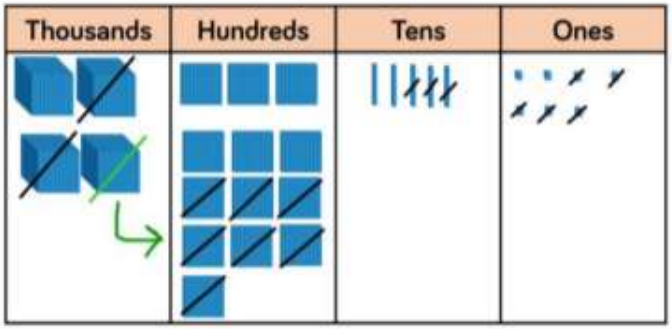
$$\begin{array}{r} {}^{3\ 1}\cancel{4}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

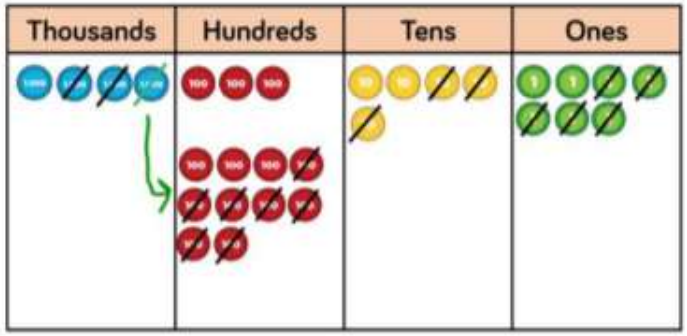
Year 4

3000

$$\begin{array}{r} \cancel{4000} + {}^{1}300 + 50 + 7 \\ -\ 2000 + 700 + 30 + 5 \\ \hline 1000 + 600 + 20 + 2 \end{array}$$

$$4{,}357 - 2{,}735 = 1{,}622$$





Language focus

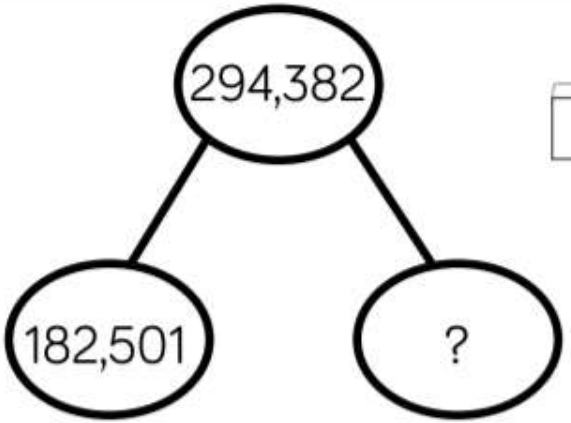
"5 ones minus 3 ones is equal to 2 ones."
"6 tens minus 2 tens is equal to 4 tens."

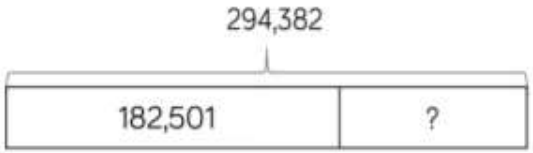
Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

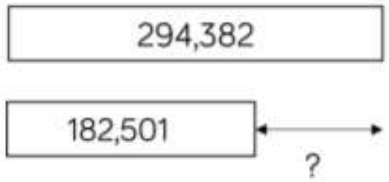
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

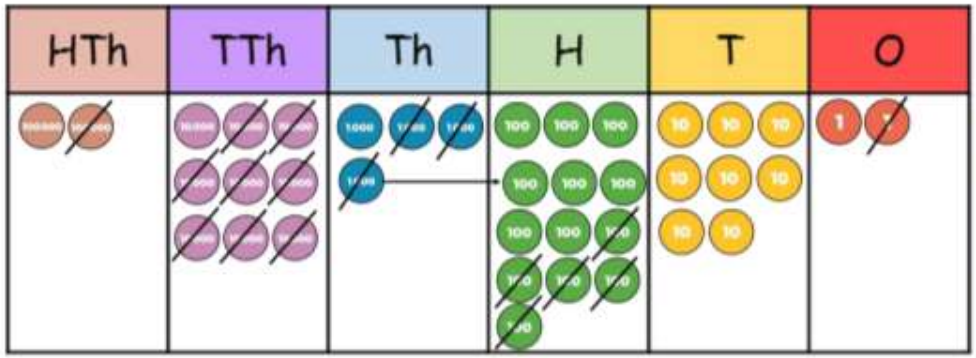
Skill: Subtract numbers with more than 4 digits







294,382 − 182,501 = 111,881



	2	9	$^{3}\cancel{4}$	$^{1}3$	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

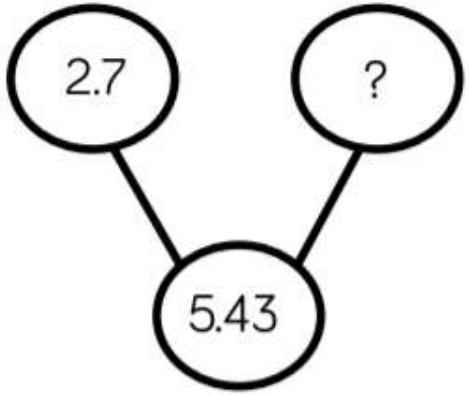
Language focus

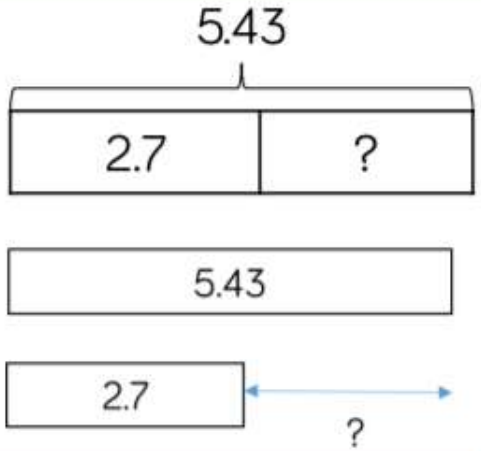
"5 ones minus 3 ones is equal to 2 ones."
"6 tens minus 2 tens is equal to 4 tens."

Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

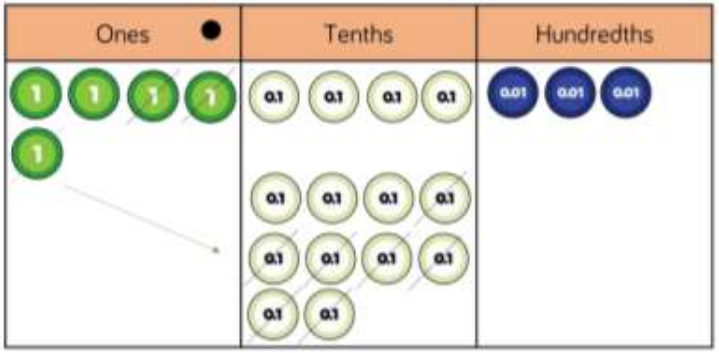
Skill: Subtract with up to 3 decimal places

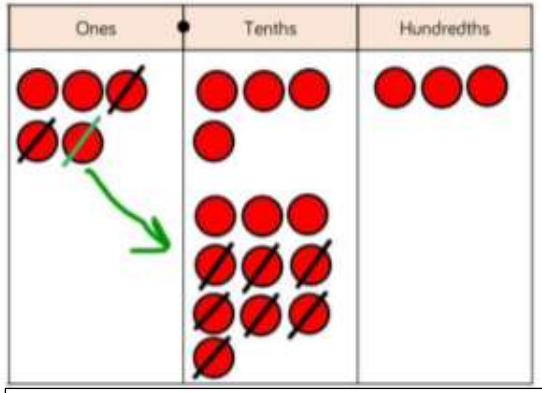




$$\begin{array}{r} {}^{4}\cancel{5}.{}^{1}43 \\ -\ 2.7 \\ \hline 2.73 \\ \hline \end{array}$$

$$5.43 - 2.7 = 2.73$$



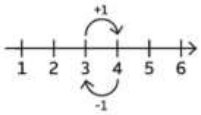
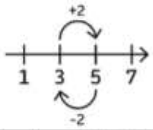
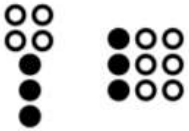
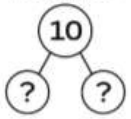
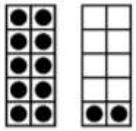
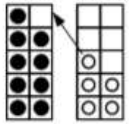
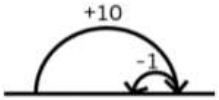


Language focus

"5 ones minus 3 ones is equal to 2 ones."
"6 tens minus 2 tens is equal to 4 tens."

Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

One More, One Less 	When we add one, we get the next counting number. When we subtract one, we get the previous counting number (e.g. 5 – 1 = 4).	**Number Neighbours: Spot the Difference** 	Adjacent numbers have a difference of 1. Adjacent odds and evens have a difference of 2. Spot number neighbours (adjacent, odds or evens) to solve subtractions of adjacent numbers (e.g. 5 – 4 = 1), of adjacent odds (e.g. 9 – 7 = 2) or adjacent evens (e.g. 6 – 4 = 2)
Two More, Two Less: Think Odds and Evens 	If we add two to a number, we go from odd to next odd or even to next even. If we subtract two from a number, we go from odd to previous odd or even to previous even.	**7 Tree and 9 Square** 	Use these visual images to remember addition and subtractions fact families that children can find tricky. For example, visualising the 7 tree helps remember that 7 – 3 = 4. Visualising the 9 square helps remember that 3 + 6 = 9.
Number 10 Fact Families 	Go beyond just recalling the pairs of numbers that add to 10. Make sure that we can also spot additions and subtractions which we can use number bonds to 10 to solve.	**Ten and A Bit** 	The numbers 11 – 20 are made up of 'Ten and a Bit'. Recognising and understanding the 'Ten and a Bit' structure of these numbers enables addition and subtraction facts involving their constituent parts (e.g. 3 + 10 = 13, 17 – 7 = 10, 12 – 10 = 2).
Five and A Bit 	The numbers 6, 7, 8 and 9 are made up of 'five and a bit'. This can be shown on hands, and supports decomposition of these numbers into their five and a bit parts (e.g. 5 + 3 = 8, 9 – 5 = 4).	**Make Ten and Then...** 	Additions which cross the 10 boundary can be calculated by 'Making Ten' first, and then adding on the remaining amount (e.g. 8 + 6 can be calculated by thinking '8 + 2 = 10 and 4 more makes 14'). The same strategy can be applied to subtractions through 10.
Know about 0 **0**	When we add 0 to or subtract 0 from another number, the total remains the same. If we subtract a number from itself, the difference is 0.	**Adjust It** 	Any addition and subtraction can be calculated by adjusting from a fact you know already, (e.g. 6 + 9 is one less than 6 + 10).
Doubles and Near Doubles 	Memorise doubles of numbers to 10, using a visual approach. Then use these known double facts to calculate near doubles and hidden doubles. Once we know 6 + 6 = 12 then 6 +7 and 5 + 7 is easy.	**Swap It**  1 + 6	When the order of two numbers being added (addends) is exchanged the total remains the same. E.g. 1 + 8 = 8 + 1. Sometimes reversing the order of the two addends makes addition easier to think about conceptually.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.

sum	
addend	addend

minuend	
subtrahend	difference

minuend	
difference	subtrahend

Multiplication and Division

The full set of multiplication calculations that pupils need to be able to solve by automatic recall are shown in the table below. Pupils must also have automatic recall of the corresponding division facts.

1 × 1	1 × 2	1 × 3	1 × 4	1 × 5	1 × 6	1 × 7	1 × 8	1 × 9	1 × 10	1 × 11	1 × 12
2 × 1	2 × 2	2 × 3	2 × 4	2 × 5	2 × 6	2 × 7	2 × 8	2 × 9	2 × 10	2 × 11	2 × 12
3 × 1	3 × 2	3 × 3	3 × 4	3 × 5	3 × 6	3 × 7	3 × 8	3 × 9	3 × 10	3 × 11	3 × 12
4 × 1	4 × 2	4 × 3	4 × 4	4 × 5	4 × 6	4 × 7	4 × 8	4 × 9	4 × 10	4 × 11	4 × 12
5 × 1	5 × 2	5 × 3	5 × 4	5 × 5	5 × 6	5 × 7	5 × 8	5 × 9	5 × 10	5 × 11	5 × 12
6 × 1	6 × 2	6 × 3	6 × 4	6 × 5	6 × 6	6 × 7	6 × 8	6 × 9	6 × 10	6 × 11	6 × 12
7 × 1	7 × 2	7 × 3	7 × 4	7 × 5	7 × 6	7 × 7	7 × 8	7 × 9	7 × 10	7 × 11	7 × 12
8 × 1	8 × 2	8 × 3	8 × 4	8 × 5	8 × 6	8 × 7	8 × 8	8 × 9	8 × 10	8 × 11	8 × 12
9 × 1	9 × 2	9 × 3	9 × 4	9 × 5	9 × 6	9 × 7	9 × 8	9 × 9	9 × 10	9 × 11	9 × 12
10 × 1	10 × 2	10 × 3	10 × 4	10 × 5	10 × 6	10 × 7	10 × 8	10 × 9	10 × 10	10 × 11	10 × 12
11 × 1	11 × 2	11 × 3	11 × 4	11 × 5	11 × 6	11 × 7	11 × 8	11 × 9	11 × 10	11 × 11	11 × 12
12 × 1	12 × 2	12 × 3	12 × 4	12 × 5	12 × 6	12 × 7	12 × 8	12 × 9	12 × 10	12 × 11	12 × 12

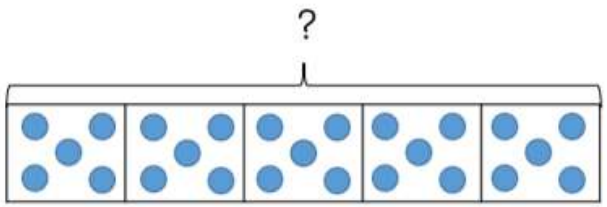
Pupils must be fluent in these facts by the end of year 4, and this is assessed in the multiplication tables check. Pupils should continue with regular practice through year 5 to secure and maintain fluency.

The 36 most important facts are highlighted in the table. Fluency in these facts should be prioritised because, when coupled with an understanding of commutativity and fluency in the formal written method for multiplication, they enable pupils to multiply any pair of numbers.

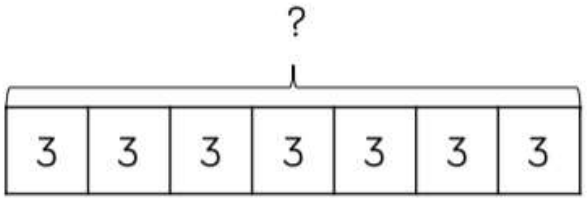
See our Times Table planner for order of teaching.

Models and Images

Bar Model

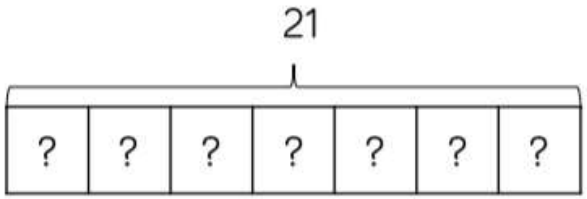


$5 \times 5 = 25$

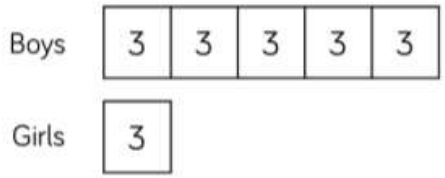


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

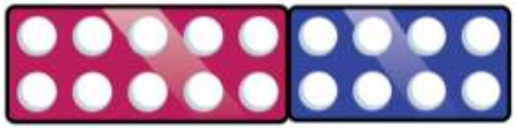
It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

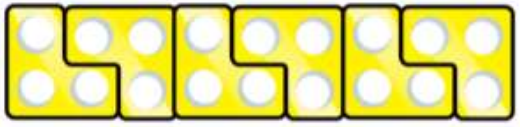
Number Shapes

$5 \times 4 = 20$
$4 \times 5 = 20$

$5 \times 4 = 20$
$4 \times 5 = 20$



$18 \div 3 = 6$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd × odd = even, odd × even = odd, even × even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

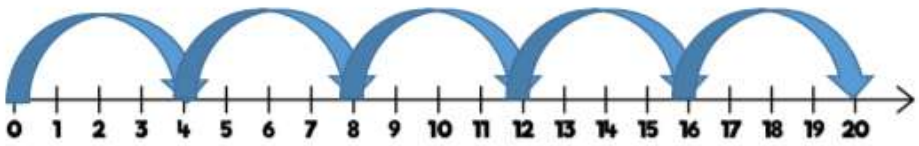
Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

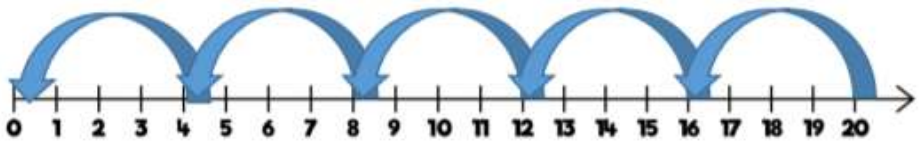
Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Lines (labelled)



$4 \times 5 = 20$
$5 \times 4 = 20$



$20 \div 4 = 5$

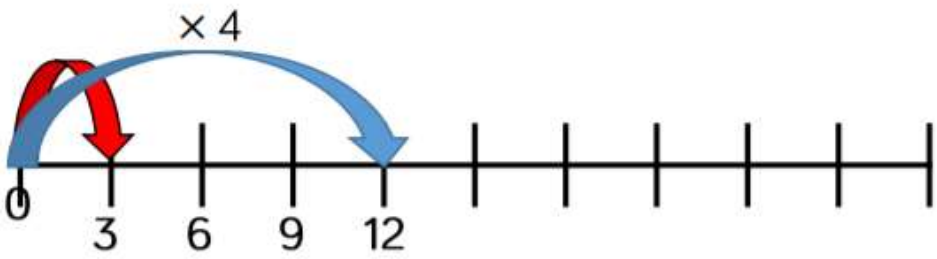
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

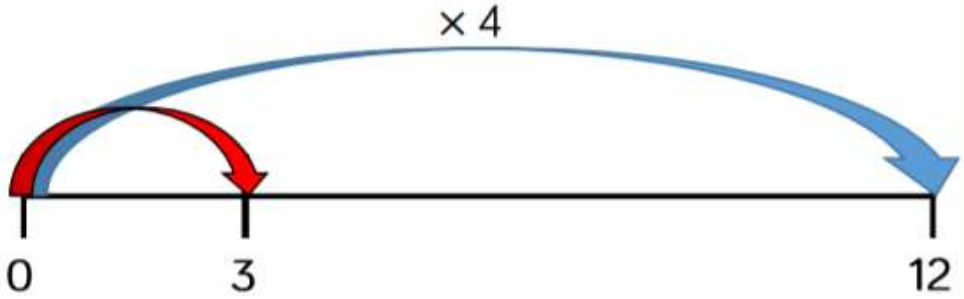
When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

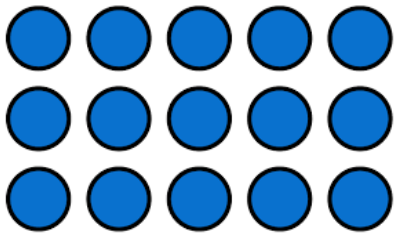
Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Arrays



3 x 5 = 15

5 x 3 = 15

5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

15 ÷ 5 = 3

15 ÷ 3 = 5

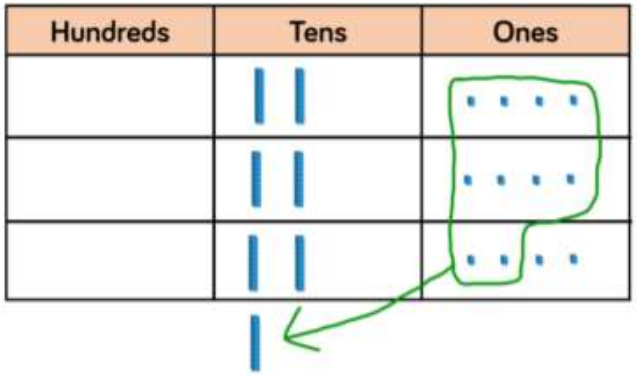
Benefits

Children can use arrays to represent multiplication facts.

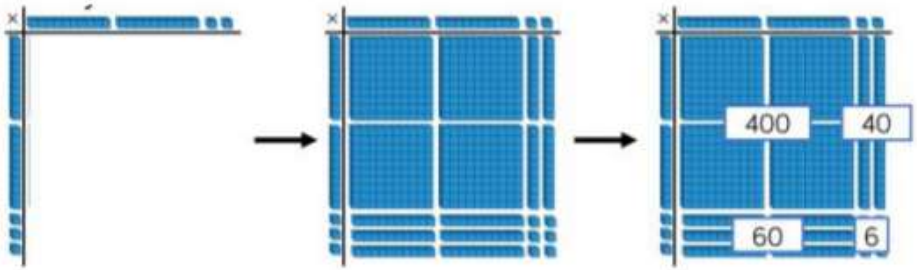
These help to make links to both repeated addition and division.

These are also a precursor to find the area of a rectangle.

Base 10/Dienes (multiplication)



$$\begin{array}{r} 24 \\ \times \quad 3 \\ \hline 72 \\ \hline 1 \end{array}$$



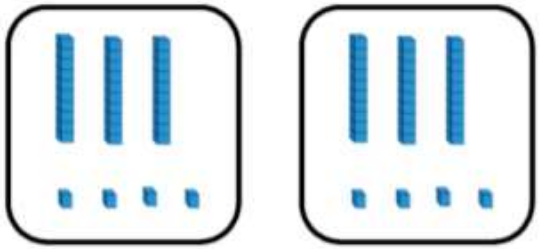
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

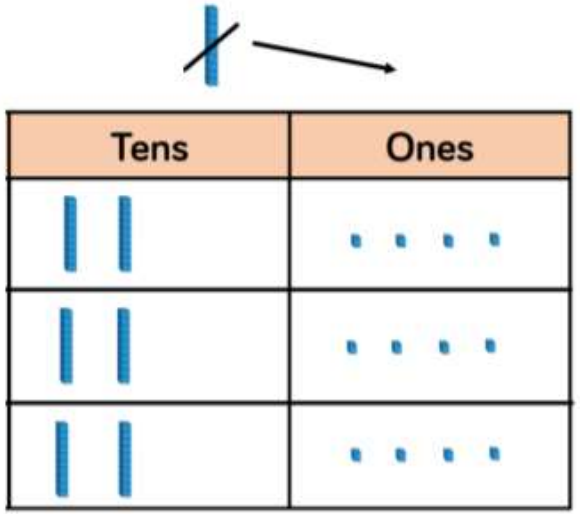
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

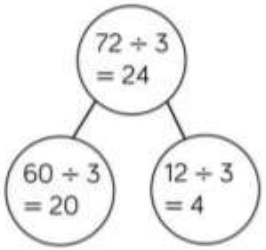
Base 10/Dienes (division)



$68 \div 2 = 34$



$72 \div 3 = 24$



Benefits

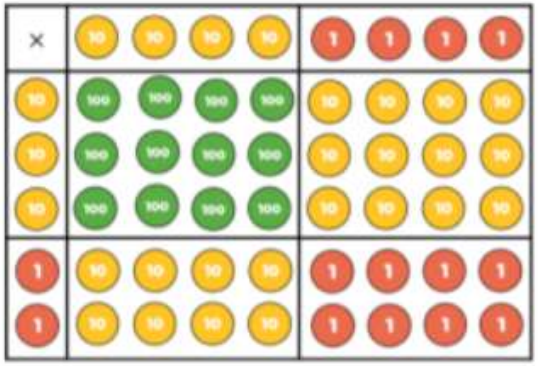
Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)

$$\begin{array}{r} 34 \\ \times\ \ 5 \\ \hline 170 \\ \hline 1\ 2 \end{array}$$



$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

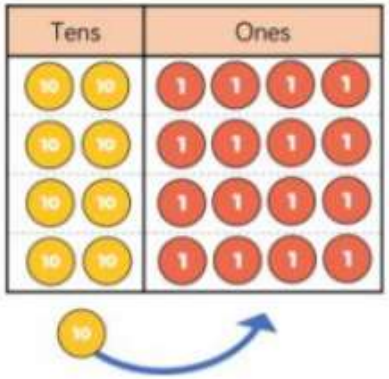
Benefits

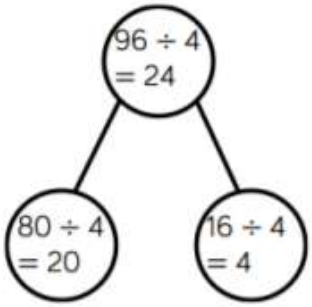
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

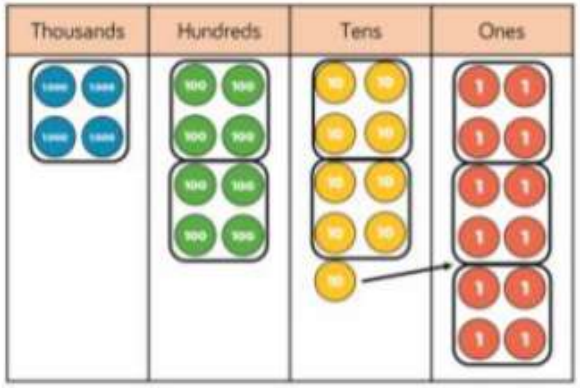
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)







$$\begin{array}{r} 1223 \\ 4\overline{)\,489^{1}2} \end{array}$$

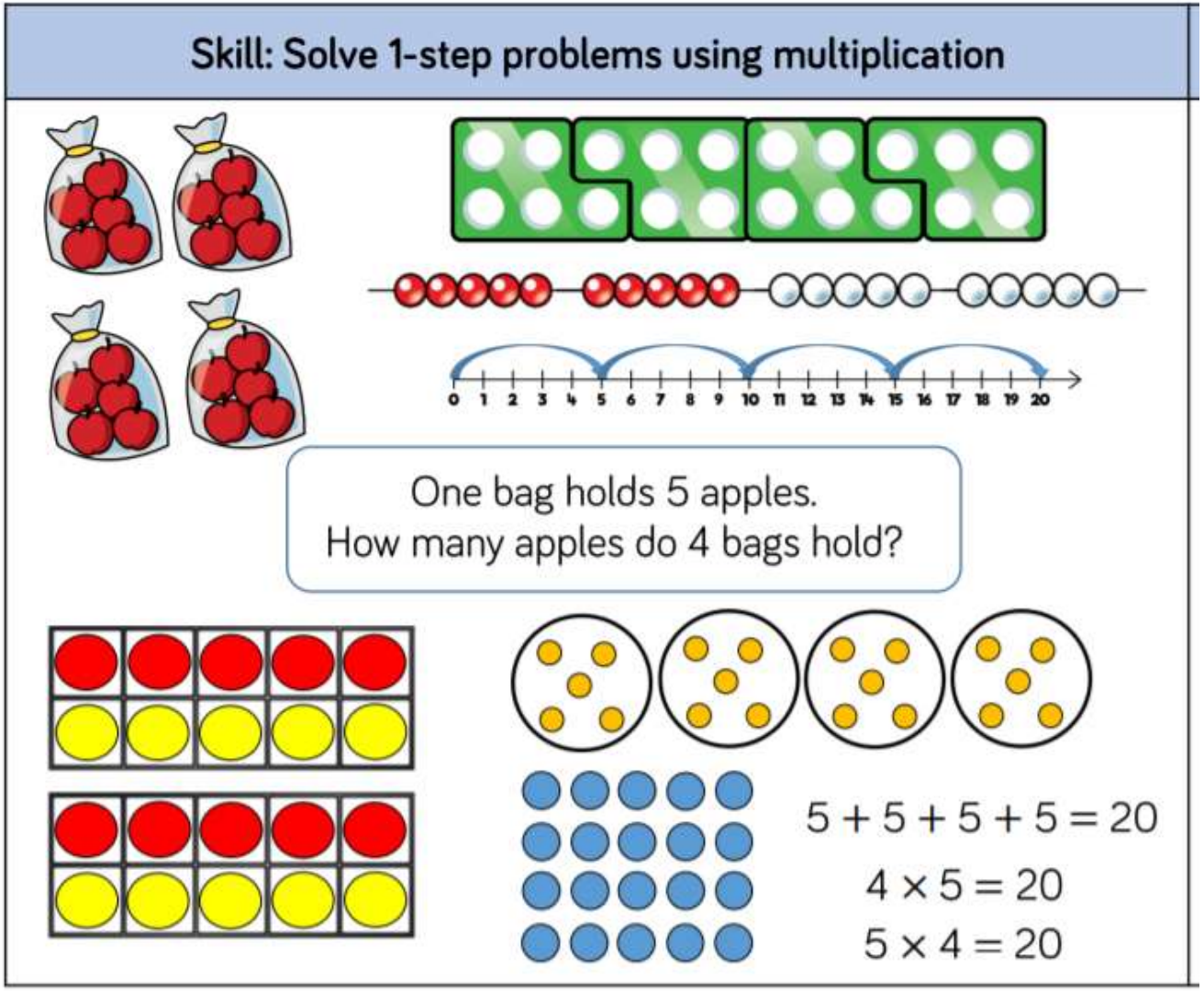
Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

MULTIPLICATION

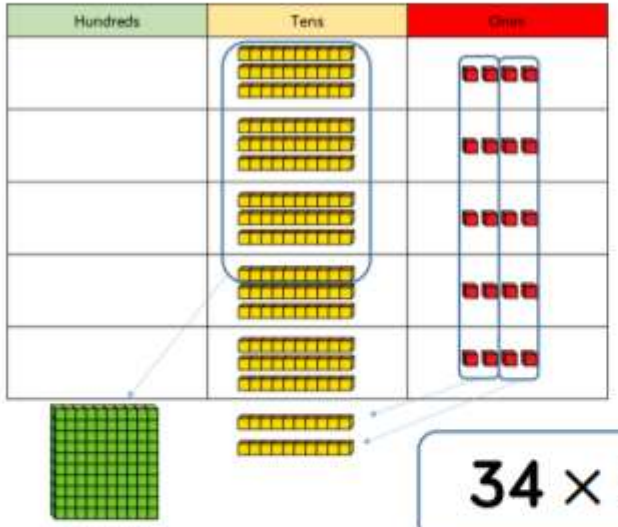


For children at the start of their multiplication journey, they should still be exposed to problems involving multiplication.

Manipulatives can be used as well as pictures for recording.

The children should also record the equation alongside.

Skill: Multiply 2-digit numbers by 1-digit numbers

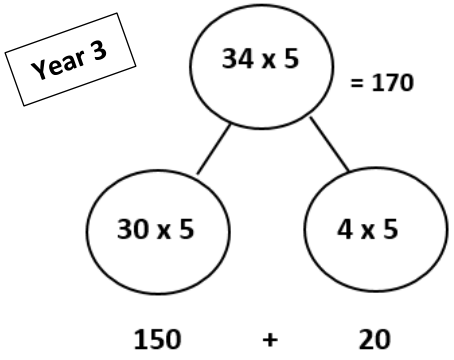


Year 4

	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

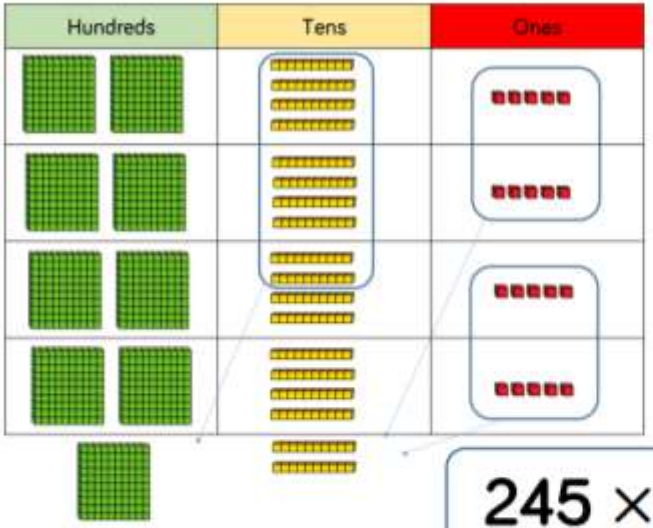
$$34 \times 5 = 170$$

Year 3



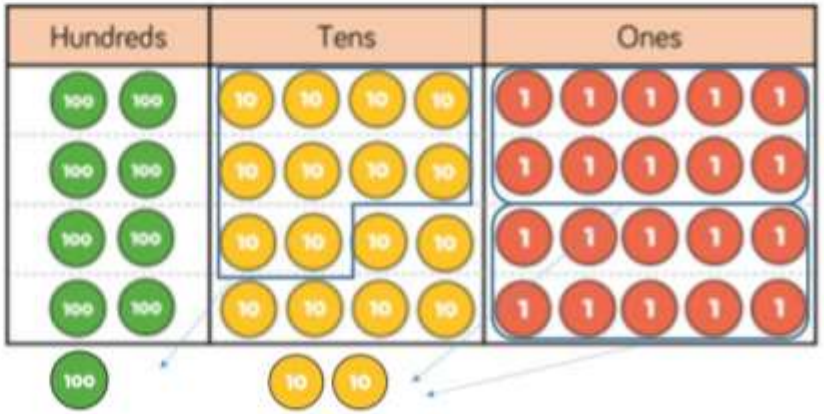
The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

Skill: Multiply 3-digit numbers by 1-digit numbers



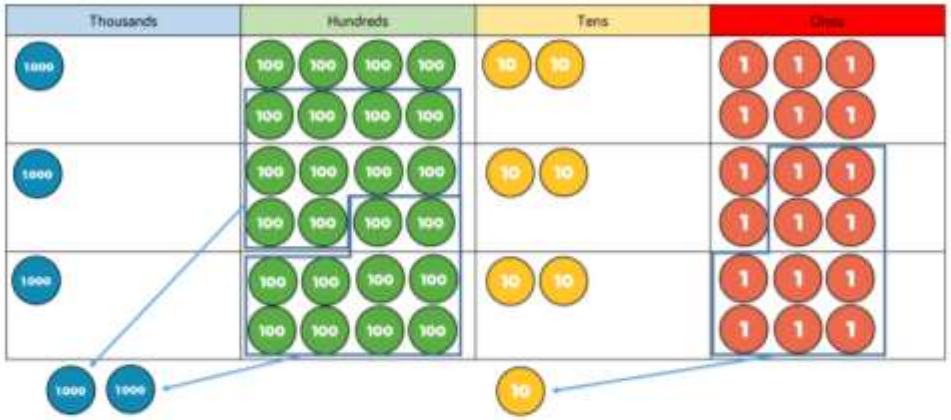
	H	T	O
	2	4	5
×			4
		2	0
	1	6	0
	8	0	0
	9	8	0

$$245 \times 4 = 980$$



Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

Skill: Multiply 4-digit numbers by 1-digit numbers



$$1{,}826 \times 3 = 5{,}478$$

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

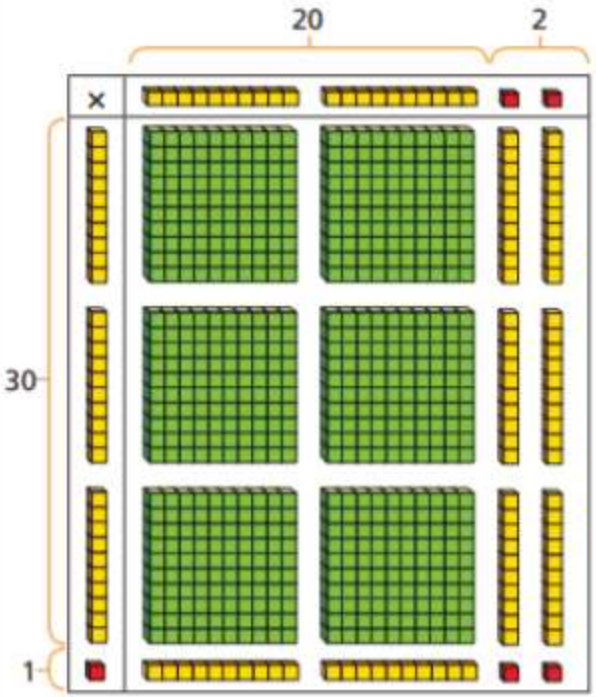
When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.
If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

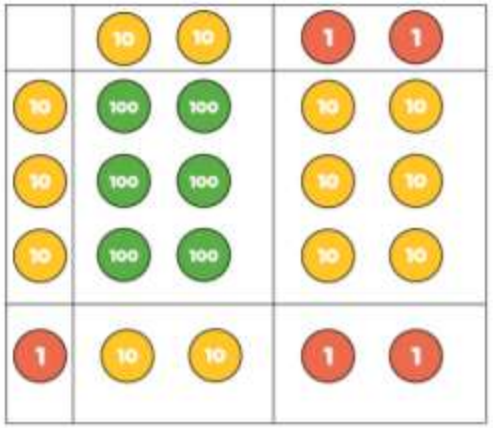
$$\begin{array}{r} 3\ 6\ 7 \\ \times \qquad 4 \\ \hline 1,4\ 6\ 8 \\ \hline 2\ \ 2 \end{array}$$

4×7 ones	= 28 ones
	= 2 tens + 8 ones
4×6 tens	= 24 tens
	= 2 hundreds + 4 tens
plus 2 more tens	= 2 hundreds + 6 tens
4×3 hundreds	= 12 hundreds
	= 1 thousand + 2 hundreds
plus 2 more hundreds	= 1 thousand + 4 hundreds

Pupils must learn that, although short multiplication can be used to multiply any number by a one-digit number, it is not always the most appropriate choice. For example, 201 x 4 can be calculated mentally by applying the distributive property of multiplication (200 x 4 = 800, plus 4 more).

Skill: Multiply 2-digit numbers by 2-digit numbers





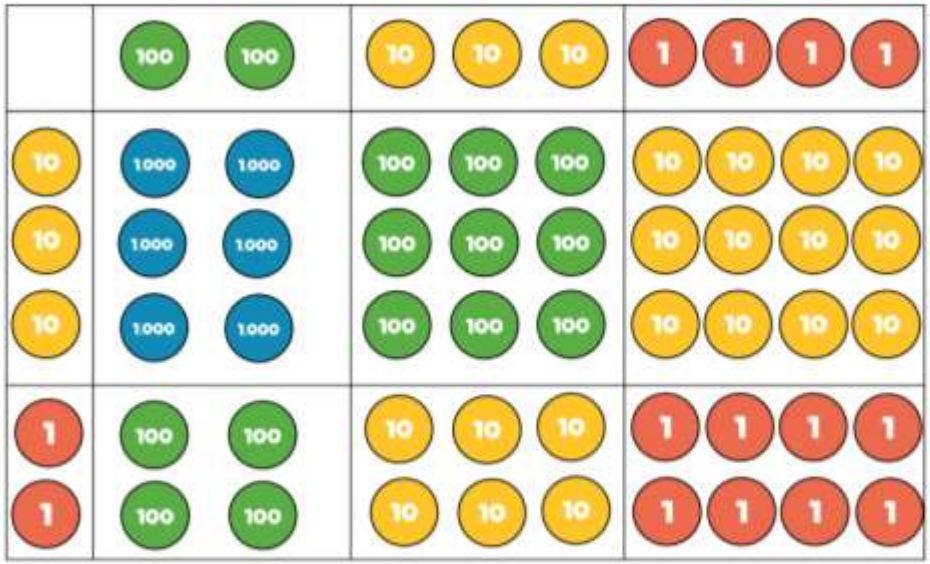
×	20	2
30	600	60
1	20	2

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

$$22 \times 31 = 682$$

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10.
The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 3-digit numbers by 2-digit numbers



Th	H	T	O
	2	3	4
×		3	2
	4	6	8
1 7	1 0	2	0
7	4	8	8

$234 \times 32 = 7{,}488$

Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.

Bottom left corner

Skill: Multiply 4-digit numbers by 2-digit numbers

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 (2)	1 (5)	9 (3)	1 (7)	2
5 (1)	4	7 (1)	8	0
7	6	6	9	2
		1		

$2{,}739 \times 28 = 76{,}692$

When multiplying 4-digits by 2-digits, children should be confident in the written method.

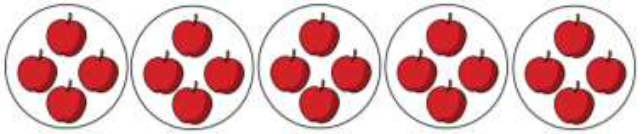
If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

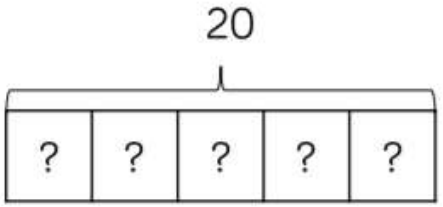
Bottom left corner

Below the line

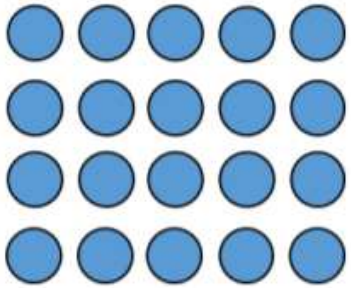
DIVISION

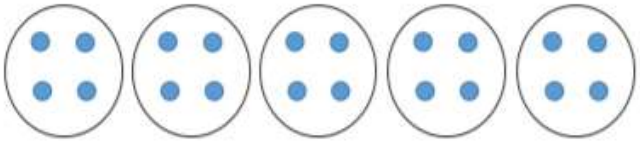
Skill: Solve 1-step problems using multiplication (sharing)





There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?

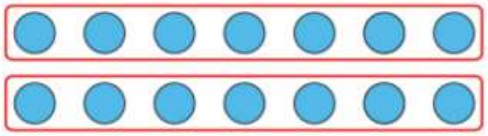




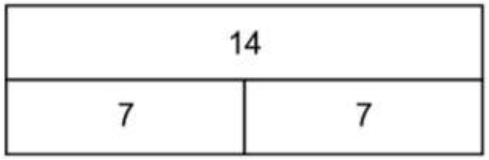
$20 \div 5 = 4$

Partitive division

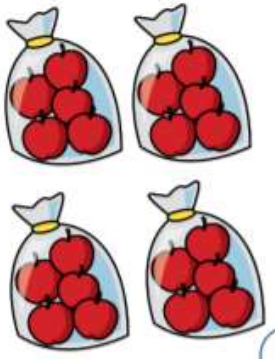
£14 is shared between 2 children. How much money does each child get?

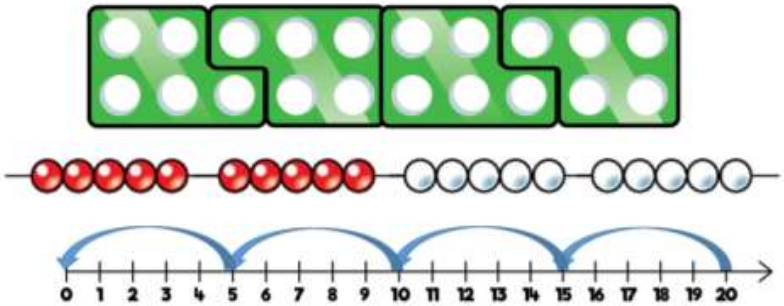


$14 \div 2 = 7$

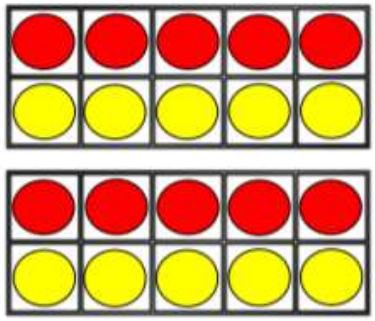


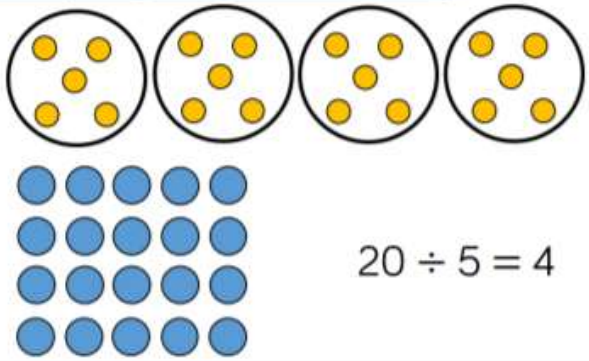
Skill: Solve 1-step problems using division (grouping)





There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?

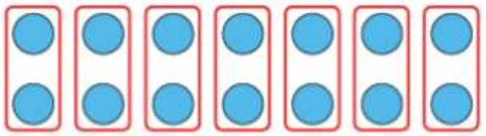




$20 \div 5 = 4$

Quotitive division

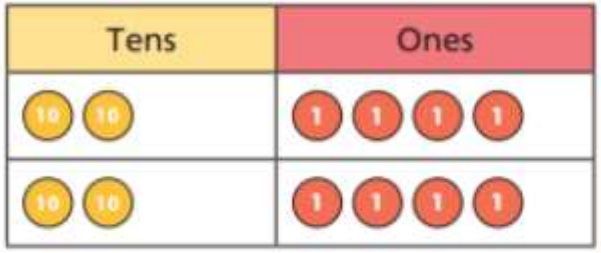
I need 14 ping-pong balls. There are 2 ping-pong balls in a pack. How many packs do I need?

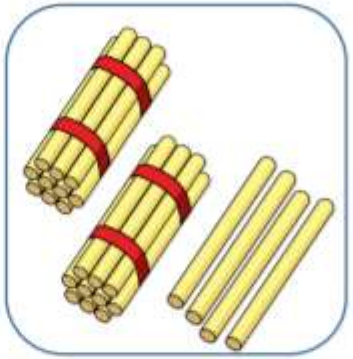


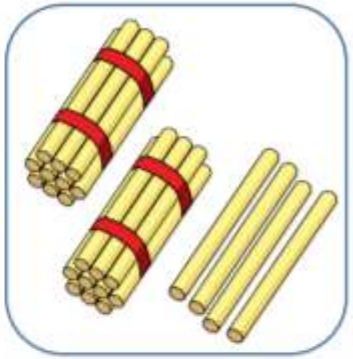
$14 \div 2 = 7$

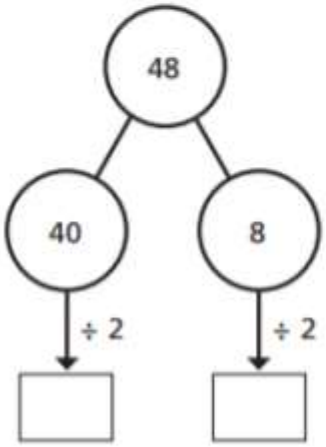
14						
2	2	2	2	2	2	2

Skill: Divide 2-digits by 1-digit (sharing with no exchange)

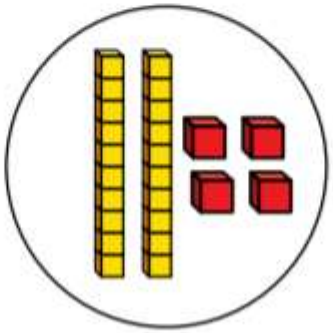


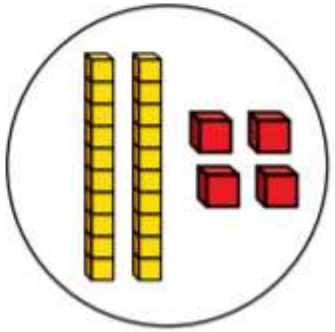






$$48 \div 2 = 24$$



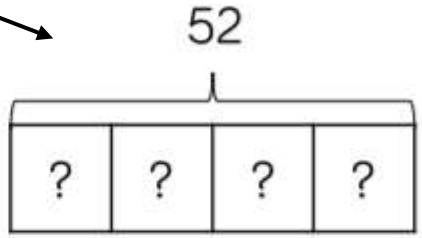


When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

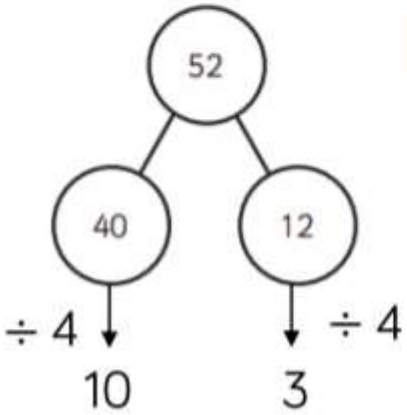
Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

Part-whole models can provide children with a clear written method that matches the concrete representation.

Skill: Divide 2-digits by 1-digit (sharing with exchange)



$$52 \div 4 = 13$$



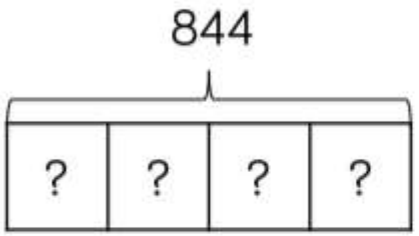
$10 + 3 = 13$

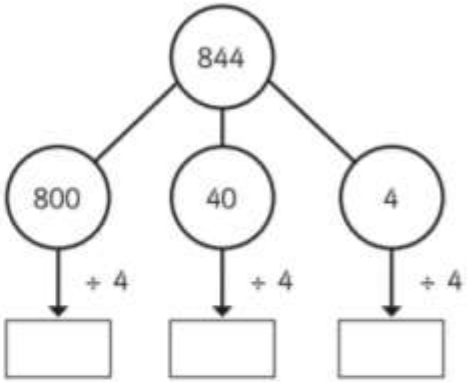
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.
Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method.

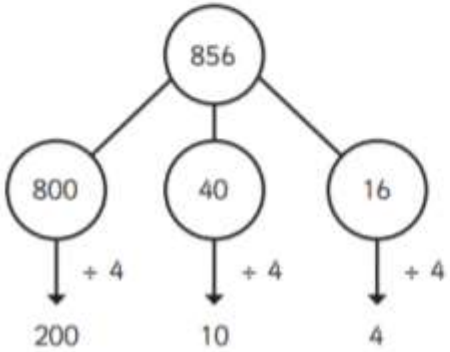
Skill: Divide 3-digits by 1-digit (sharing)

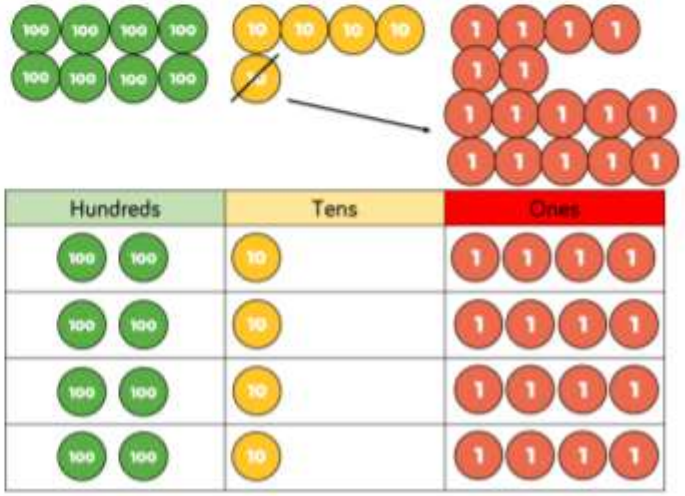
844 ÷ 4 = 211





844 ÷ 4 = 211

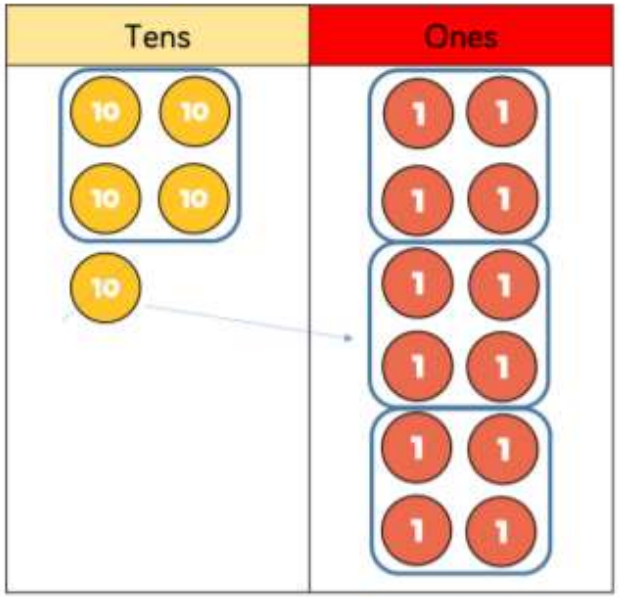




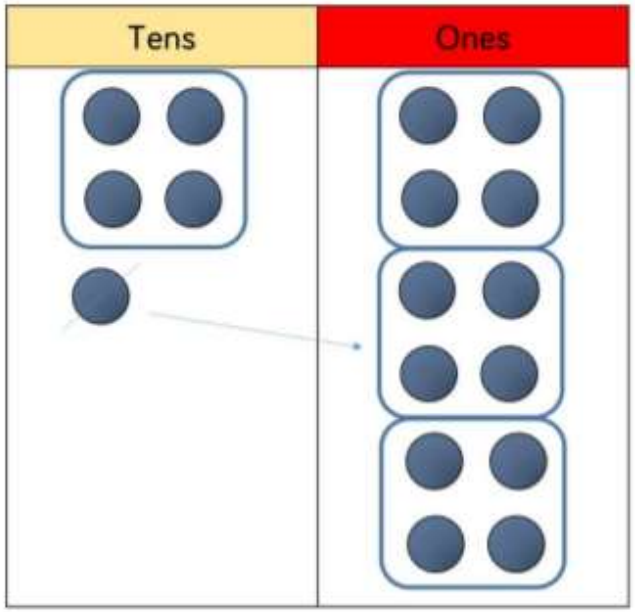
Children can continue to use place value counters to share 3-digit numbers into equal groups.
Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows.
This method can also help to highlight remainders.
Flexible partitioning in a part-whole model supports this method.

Year 5 +

Skill: Divide 2-digits by 1-digit (grouping)



		1	3	
	4	5	12	



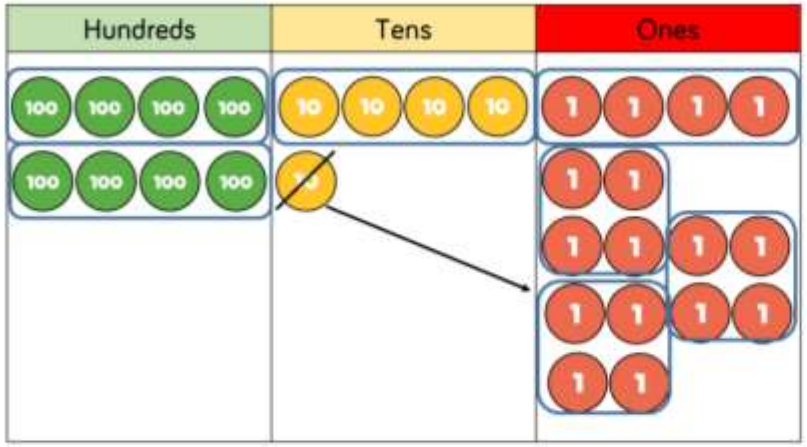
$$52 \div 4 = 13$$

When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

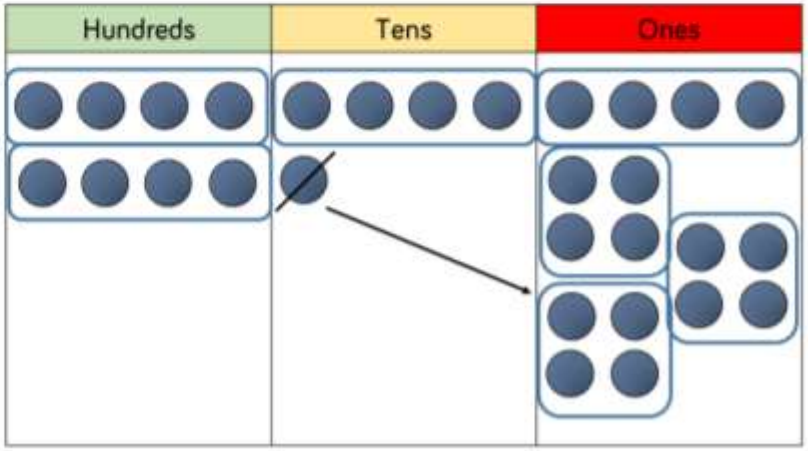
Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

Skill: Divide 3-digits by 1-digit (grouping)



		2	1	4
	4	8	5	$^{1}6$

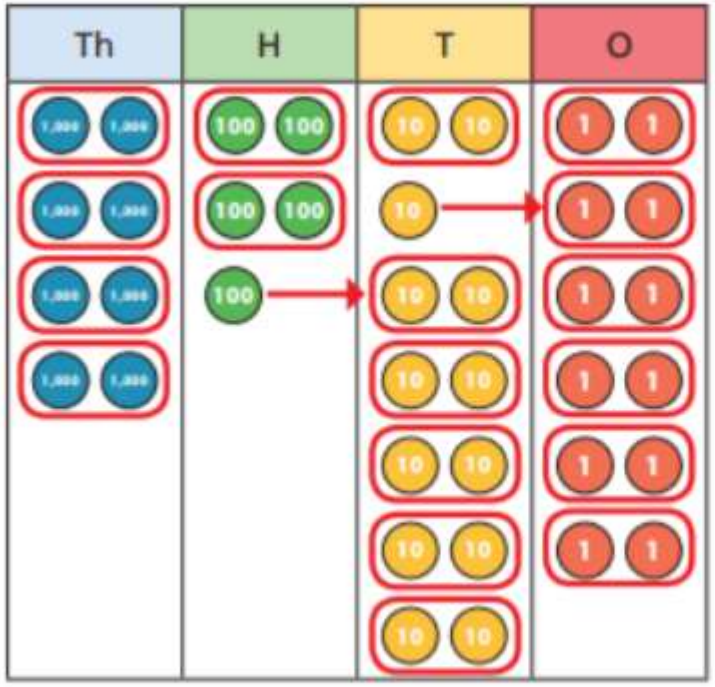


$$856 \div 4 = 214$$

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

Skill: Divide 4-digits by 1-digit (grouping)



	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

$$8{,}532 \div 2 = 4{,}266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Skill: Divide multi-digits by 2-digits (long division)

		0	3	6
1	2	4	3	2
	–	3	6	0
			7	2
	–		7	2
				0

(×30)

(×6)

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$$432 \div 12 = 36$$

	0	4	8	9
15	7	3	3	5
–	6	0	0	0
	1	3	3	5
–	1	2	0	0
		1	3	5
–		1	3	5
				0

(×400

(×80)

(×9)

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

$$7{,}335 \div 15 = 489$$

15	30	45	60	75	90	105	120	135	150

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi digits by 2-digits (long division)

$$372 \div 15 = 24 \text{ r}12$$

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	–	3	0	0	
			7	2	
	–		6	0	
			1	2	

$$372 \div 15 = 24\frac{4}{5}$$

$= 24.8$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction.
This will depend on the context of the question.

Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor